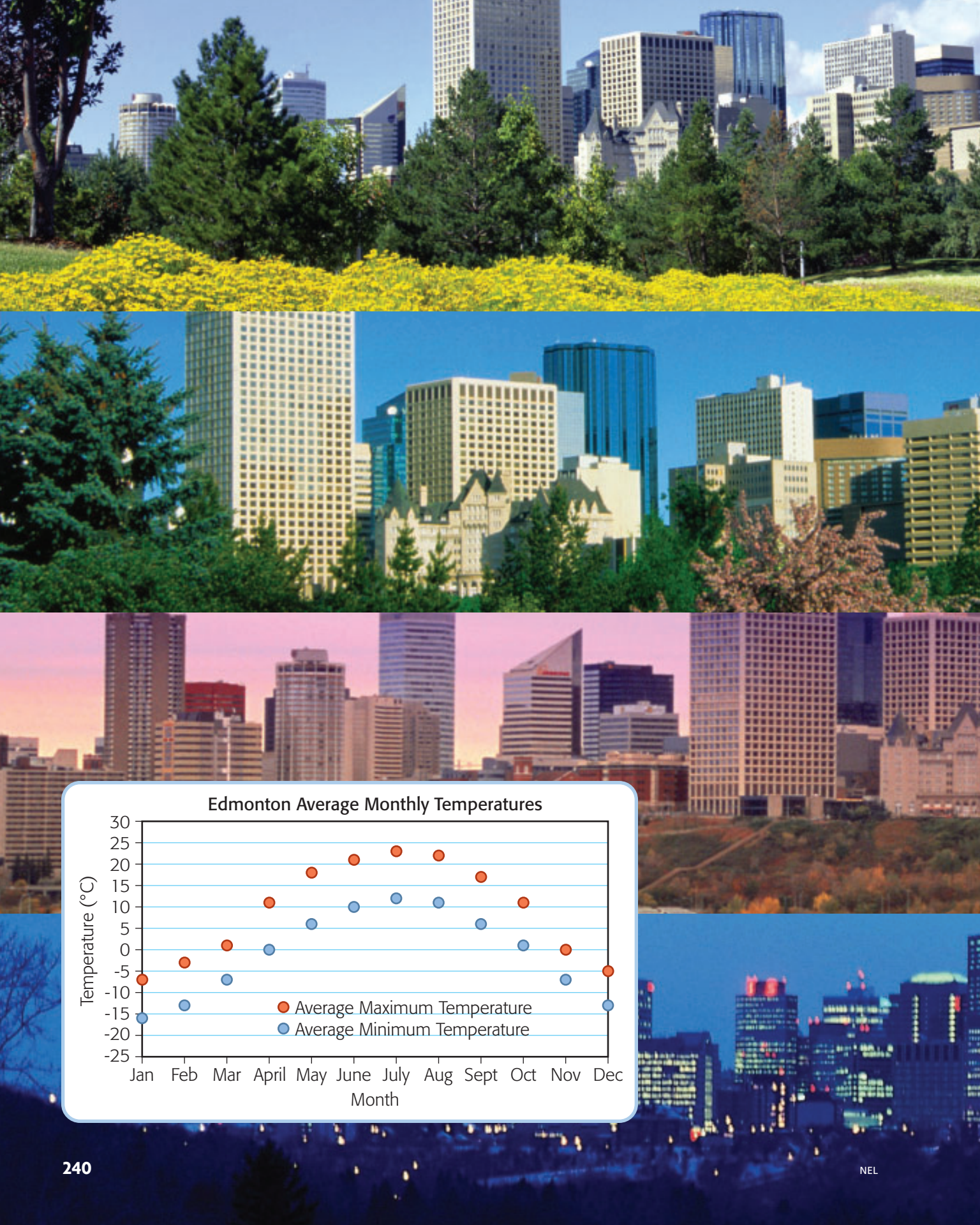

Edmonton Average Monthly Temperatures
Temperature (°C)
30
25
20
15
10
5
0
-5
-10
-15
-20
-25
Jan
Feb
Mar
April
May
June
July
Aug
Sept
Oct
Nov
Dec
Month
Average Maximum Temperature
Average Minimum Temperature

Chapter

6

Integers

GOAL

You will be able to

- represent multiplication and division of integers using concrete materials, drawings, and number lines
- record the multiplication and division of integers symbolically
- solve problems that involve addition, subtraction, multiplication, and division of integers

◀ How do you think the average monthly minimum and maximum temperatures were calculated?

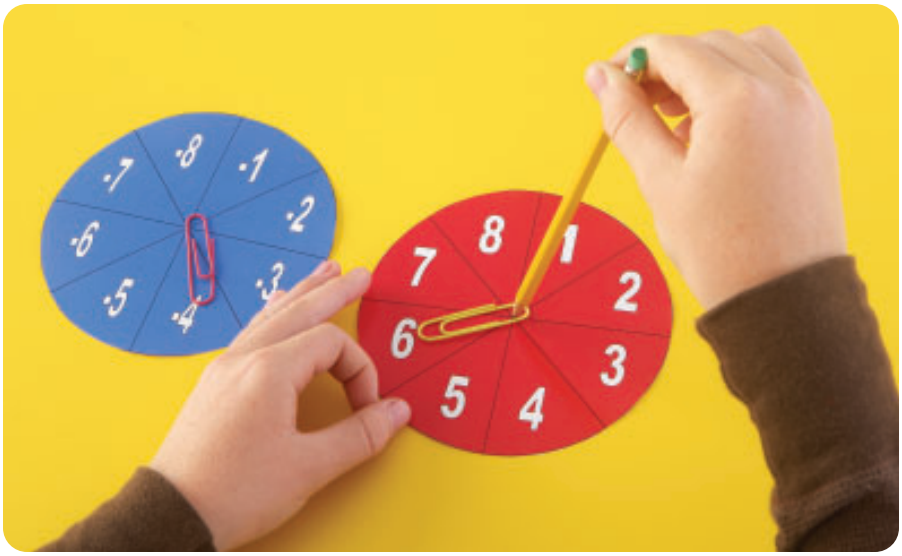
Getting Started

YOU WILL NEED

- a red spinner and a blue spinner, each divided into eighths
- two paper clips
- red and blue counters
- number lines
- red and blue coloured pencils

Spinning Numbers

Elena has a red spinner that shows positive numbers and a blue spinner that shows negative numbers. Each spinner is divided into eight equal sections. She spins one spinner and records the number. She spins the other spinner and records the number.



What are the greatest and least sums and differences possible?

A. Spin both spinners. Record the sum and the difference in a chart like the one below. Repeat this nine more times.

Positive number (red)	Negative number (blue)	Sum	Difference

B. Does the order in which you add the numbers affect the sum? Explain.

C. Does the order in which you subtract the numbers affect the difference? Explain.

D. Can you get a sum of 0? Can you get a difference of 0? Explain.

E. What is your greatest sum?
What is the greatest sum you could get?

F. What is your least sum? What is the least sum you could get?

G. What is your greatest difference?
What is the greatest difference you could get?

H. What is your least difference?
What is the least difference you could get?

What Do You Think?

Decide whether you agree or disagree with each statement.
Be ready to explain your decision.

1. 5×4 can be thought of as the repeated addition $4 + 4 + 4 + 4 + 4$. You can use this strategy to multiply any two integers.

2. You can divide 20 by 4 by subtracting 4 repeatedly from 20 and counting the number of subtractions needed to get a remainder of 0.

$20 - 4 = 16$
$16 - 4 = 12$
$12 - 4 = 8$ — $20 \div 4 = 5$
$8 - 4 = 4$
$4 - 4 = 0$

You can use this strategy to divide any two integers.

3. The product of two integers is probably always greater than their quotient.

6.1 Integer Multiplication

YOU WILL NEED

- red and blue counters

GOAL

Use patterns to predict the products of integers.

LEARN ABOUT the Math

Guy and Elena thought they could use coloured counters to model integer multiplication.

How can you determine the product of two integers?

Communication | *Tip*

- A positive integer may be written without a $+$ sign in front of it.
- Use brackets to separate the sign of a negative integer from any operation that comes immediately before it in a mathematical expression. For example, write $-3 + (-4)$ to separate the sign of -4 from the addition operation.
- Instead of using a $\times$ sign, you can use brackets to separate the factors in a product. For example, 3×5 and $3(5)$ represent the same product.

Example 1 | Modelling positives × negatives

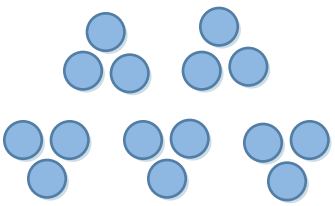
Use integer counters to model products of the form (+) × (−).

Guy's Solution

3 x (-2)

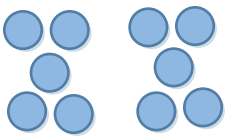
3 x (-2) = -6

5 x (-3)



5 x (-3) = -15

2 x (-5)



2 x (-5) = -10

The examples I used show that (+) x (-) = (-).

I tried a few examples of $(+) \times (-)$.

I knew that a product tells the number of identical groups of counters to combine. I used the first factor to tell the number of groups and the second factor to tell the number of negative counters in a group.

I can use this strategy to model any product of $(+) \times (-)$ as groups of negative counters.

Example 2 | Modelling negatives × positives

Calculate -2×3.

Vanessa's Solution: Using a property of multiplication

$-2 \times 3 = 3 \times (-2) = -6$
I think $-2 \times 3 = -6$.

I remembered that, when you multiply whole numbers, the order does not matter. I assumed that would be true for integers.

Kaitlyn's Solution: Using a pattern

$3 \times 3 = 9$ (−3)
$2 \times 3 = 6$ (−3)
$1 \times 3 = 3$ (−3)
$0 \times 3 = 0$ (−3)
$-1 \times 3 = -3$ (−3)
$-2 \times 3 = -6$

I used a multiplication pattern. My pattern shows that, when you decrease the first factor by 1, the product decreases by 3.

I continued this pattern until the first factor was -2.

Joseph's Solution: Using a property of integers

$-2 \times 3 = 0 - 2 \times 3$
$= 0 - 6$
$= -6$
$-2 \times 3 = -6$

I knew that -2 means $0 - 2$, so I assumed that -2×3 is the same as $0 - 2 \times 3$.

Example 3 | Using reasoning to multiply two negatives

Calculate $-2(-3)$.

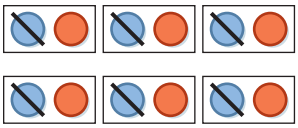
Elena's Solution: Using a multiplication pattern

$3(-3) = -9$ (+3)
$2(-3) = -6$ (+3)
$1(-3) = -3$ (+3)
$0(-3) = 0$ (+3)
$-1(-3) = 3$ (+3)
$-2(-3) = 6$

Guy told me that, when you multiply $(+) \times (-)$, the product is negative. I used a multiplication pattern that shows that, each time the first factor is decreased by 1, the product increases by 3.

Mark's Solution: Using counters and a property of integers

$-2 \times (-3) = 0 - 2 \times (-3)$



$-2(-3) = 6$

-2 means $0 - 2$, so you can say that $-2 \times (-3)$ is the same as $0 - 2 \times (-3)$.

I represented 0 with 2 groups, each containing 3 0-pairs. Then I subtracted 2 groups of 3 blue counters.

Reflecting

A. Why does it make sense that the products in each pair are opposites?

2×3 and -2×3

-2×3 and $-2 \times (-3)$

B. How can you predict the sign and value of the product of two integers? Summarize your answer using the chart.

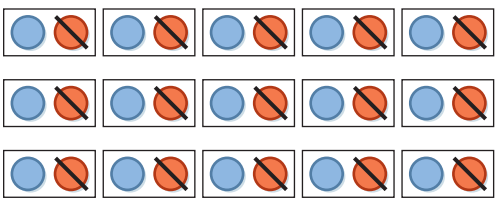
×		Second integer	
		+	−
First integer	+		
	−		

WORK WITH the Math

Example 4 | Modelling negatives × positives

Use counters to show why $-3(5) = -15$.

Solution



Think of $-3(5)$ as $0 - 3(5)$.
This means you have to remove 3 groups of 5 positive red counters from 0.

So make 3 groups of 5 0-pairs and remove the red counters.

Example 5 | Determining factors of an integer

Represent 6 as a product of two integers in as many ways as possible.

Solution

6 can be factored as

1×6 and 6×1
$-1 \times (-6)$ and $-6 \times (-1)$
2×3 and 3×2
$-2 \times (-3)$ and $-3 \times (-2)$

If the product is positive, either both factors are positive or both are negative.

Example 6 | Creating a context requiring multiplication

Describe a situation that requires multiplying two integers to answer a question.

Solution

The water level in a tube is dropping at a rate of 3 mm/s.

a) How far will it drop in 10 s?

You can think of the rate at which the water level dropped as a negative integer (−3 mm/s).

$10 \times (-3) = -30$ mm

b) If the height of the water is now 150 mm, what was the height 10 s ago?

Then you can think of the time as negative if you think about what happened in the past.

$150 + (-10) \times (-3) = 150 + 30$
$= 180$ mm

A Checking

1. Write an expression that has the same product.

a) $3(-4)$ **c)** $-2(-7)$
b) $-3(1)$ **d)** $-5(-4)$

2. Multiply the integers in each expression using counters.

a) $2 \times (-5)$ **b)** $-4(-3)$ **c)** $-6(2)$

B Practising

3. Calculate.

a) -3×4

b) $4(-2)$

c) $-5(-5)$

d) -8×2

4. Represent each using a model.

a) $5 \times (-2)$

b) -5×2

c) $-5 \times (-2)$

5. Write the integer multiplication represented by each counter model.

a)

b)

c)

d)

6. Calculate.

a) -2×10

b) $-10 \times (-2)$

c) $2 \times (-8)$

d) $-8 \times (-2)$

e) $5 \times (-6)$

f) -5×6

7. Which two integers would make each true?

a) The sum of the integers is 23 less than the product.

b) The sum of the integers is 28 more than the product.

c) The sum of the integers is 73 more than the product.

8. Complete the following.

a) $6 \times \square = (-3) \times 4$

b) $\square \times (-3) = 5 \times \square$

c) $\square \times 4 = (-6) \times \square$

d) $\square \times (-2) = (-6) \times \square$

9. Explain why the product of any two integers is the same as the product of their opposites.

10. A deck of cards has two cards each of the integers from -5 to 5. Suppose you are dealt two cards from the deck and multiply the numbers on those cards.

a) Which two cards would give you the greatest product?

b) Which two cards would give you the least product?

Reading Strategy

Questioning

What questions can you ask to help you understand the problem?

11. Replace the ■ with =, <, or > to make each statement true.

a) $-1 \times (-2)$ ■ -4

b) $4 \times (-5)$ ■ -20

c) $-2 \times (-4)$ ■ 7

d) $3 \times (-1)$ ■ -2

e) $-6 \times (-2)$ ■ 11

f) -4×2 ■ -7

12. **a)** Write -16 as a product of two integers in as many different ways as possible.

b) Write 16 as a product of two integers in as many different ways as possible.

13. The product of three integers is –24. Name five possibilities for the three integers.

14. Write each as a product.

a) $-9 + (-10) + (-11)$

b) $(-17) + (-8) + (-25)$

c) $12 - (-7) - 20$

d) $(-12) + (-18) - 10$

15. Jasmine has 50 shares of a company. The value of each share went down by $2 today. Express the total change in value of Jasmine's shares as an integer calculation.

16. **a)** Explain why each of these equations is true.

A. $7 \times (-2) = -7 \times 2$

B. $(-2) \times (-7) = 2 \times 7$

b) Create a situation in which each expression could be used to solve a problem.

17. How can you predict the sign of each product without actually calculating it?

a) $-3 \times (-2) \times 4$

b) $4 \times (-5) \times 6$

18. Each pattern is based on multiplying integers. Complete each pattern and write a rule for the pattern.

a) 1, −3, 9, −27, 81, __, __, __

b) −3, 6, −12, __, __, __

19. You multiplied four integers together, and the answer was negative. What do you know about the signs of the integers?

6.2 Using Number Lines to Model Integer Multiplication

YOU WILL NEED

- Number Lines

GOAL

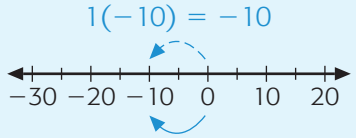
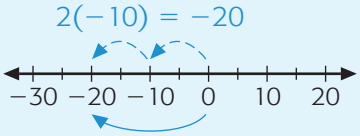
Use a pictorial model to represent integer multiplication.

LEARN ABOUT the Math

Sanjev made a movie for his media class. Now he must edit it. He can rewind or fast-forward through the movie at different speeds. The speed is measured in frames per second (fps). He resets the counter to 0 before he advances or rewinds the movie.

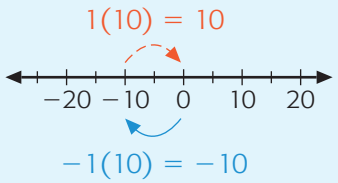
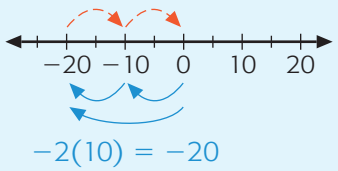
? How can Sanjev name the frame he will reach after rewinding or fast-forwarding for a given amount of time?

A. The frame counter is set to 0. You rewind at 10 fps. Complete the following chart showing the frame counter for 1 s, 2 s, 3 s, and 4 s. Use a dotted arrow to show each jump and a solid arrow to show the result.

Rewind time (s)	Number line model	Ending frame	Equation
1		−10	1(−10) = −10
2		−20	2(−10) = −20
3			
4			

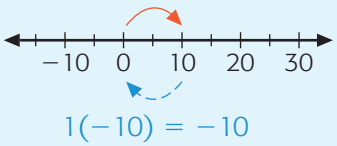
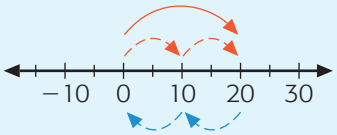
B. Predict the ending frames if you rewind for 10 s, 20 s, and 30 s, respectively.

C. You fast-forward at 10 fps and notice that the counter shows 0 when you stop. Complete the following chart showing the starting frame number for each fast-forward time.

Fast-forward time (s)	Number line model	Starting frame	Equation
1		−10	−1(10) = −10
2		−20	−2(10) = −20
3			
4			

D. Predict the starting frame if you had fast-forwarded for 10 s, 20 s, and 30 s, respectively, and arrived at frame 0.

E. You rewind at 10 fps and notice that the counter shows 0 when you stop. Complete the following chart showing the starting frame number for rewind time.

Rewind time (s)	Number line model	Starting frame	Equation
1	$-1(-10) = 10$  $1(-10) = -10$	10	$-1(-10) = 10$
2	$-2(-10) = 20$ 	20	
3			
4			

F. Predict the starting frame if you had rewound for 10 s, 20 s, and 30 s, respectively, and arrived at frame 0.

Reflecting

G. In parts C and E, why does it make sense to treat the time as negative when you write the multiplication equation for the starting frame?

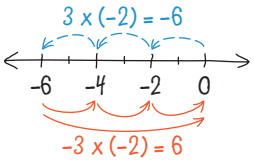
H. How would you decide whether to use counters or a number line to represent an integer multiplication?

WORK WITH the Math

Example 1 | Modelling an integer product

Multiply $-3 \times (-2)$ using a number line model.

Mark's Solution



$-3 \times (-2)$ means the same as $0 - 3 \times (-2)$, which is the opposite of $3 \times (-2)$.

To show $3 \times (-2)$, I drew 3 dotted blue arrows going left from 0, with 2 units to each arrow. The arrows stop at -6.

To show its opposite, $-3 \times (-2)$, I drew 3 solid red arrows going right from -6 back to 0. These arrows go to the right 6 units. So the answer must be 6, which is the opposite of -6.

Example 2 | Using integers to solve distance problems

Kenji walks to the west at 80 m/min.

a) **Where will he be after 5 min?**

b) **Suppose that Kenji walked to the west for 18 min. How far must he walk to return to his starting position? In which direction must he walk?**

Michel's Solution

I knew that 80 m/min means 80 m every minute.
I imagined a number line where my starting point was 0.
I drew my number line with West negative and East positive.

a) $5 \times (-80) = -400$
Kenji will be 400 m to the west.

I multiplied $5 \times 80 = 400$.
I knew that if he was walking to the west, he would end up west of 0 and that had to be negative.

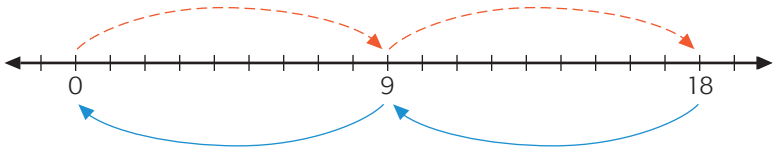
b) $18 \times (-80) = -1440$
Kenji must walk 1440 m to the east.

I multiplied $18 \times 80 = 1440$.
If he walked to the west to get somewhere, he must have started somewhere to the east of that place.

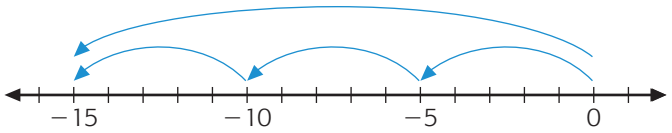
A Checking

1. Write the multiplication sentence that the blue arrows in each model represents.

a)



b)



2. Dario is on a cycling trip. He started at 0 km.
He is now at position 20 h × (−20 km/h).
When did he reach each of the following positions?
Draw a number line to show how you got your answer.

a) 10 h × (−20 km/h)

b) 8 h × (−20 km/h)

c) 0 km

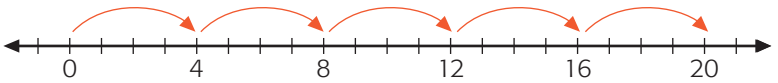
B Practising

3. Model $-4 \times (-3)$ on a number line. Calculate the product. Explain what you did.

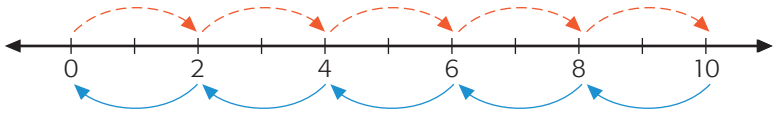
4. Use a number line to show how to determine $(-4)(-7)$.

5. Write each as a multiplication and then calculate the result.

a)

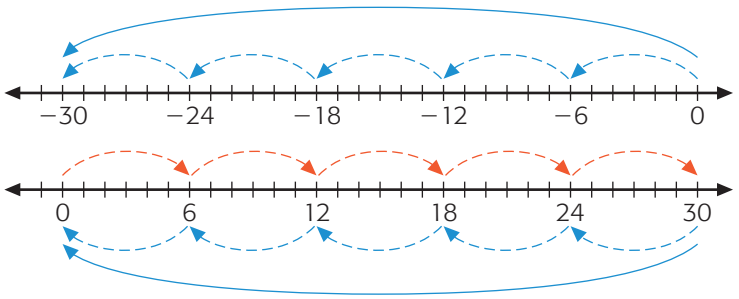


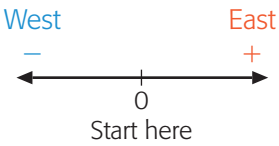
b)



6. a) Write the multiplication equation modelled by each number line diagram.

b) Why do the products represented by each diagram have the same value?





7. Write an integer multiplication sentence for each description.

a) Tyler rode a bus to the west for 4 h at 100 km/h.

b) Jenna babysat for 3 h, earning $5/h.

c) The temperature fell 2 °C a day for 6 days.

8. Multiply.

a) $0 \times (-30)$

b) $7(-20)$

c) $-4(-20)$

d) $-15(4)$

e) $-6(-30)$

f) $-20(-50)$

9. **a)** Determine the greatest product you can form using any pair of numbers from this list. Show how you know.
$-20, -10, 0, 10, 15$

b) Describe a problem situation for which the greatest product might be a solution.

10. The product of two integers is between -20 and -25. Give five possible pairs of integers for which this is true.

11. Explain how you might use a number line to solve $-9 \times \square = 108$.

12. Each pattern is based on multiplication. Fill in the next three terms and explain the pattern rule.

a) $-20, 100, -500, \_\_, \_\_, \_\_$

b) $5, -55, 605, \_\_, \_\_, \_\_$

13. Multiply.

a) $-5 \times 3 \times (-8)$

b) $-10 \times 2 \times (-5) \times (-6)$

14. The product of five different integers is -80.

a) What is the least possible sum of these integers?

b) What is the greatest possible sum?

c) Is it possible for the product of four different integers to be -80? Explain.

15. Use a number line model to show that these products are equivalent.

a) -12×10 and $12 \times (-10)$

b) $-15 \times (-20)$ and 15×20

16. **a)** How could you use a number line model to explain why $(-) \times (+) = (+) \times (-)$?

b) How could you use a number line model to explain why the product of two negative numbers is positive?

Mid-Chapter Review

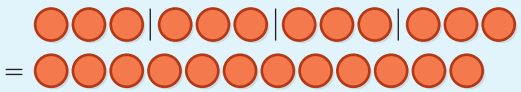
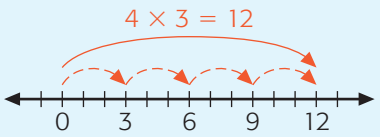
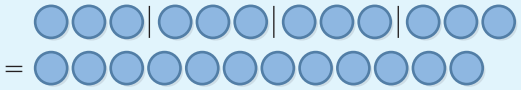
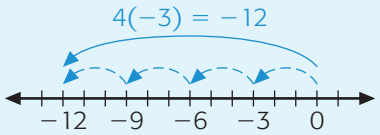
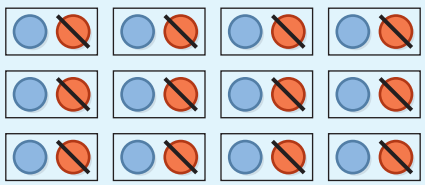
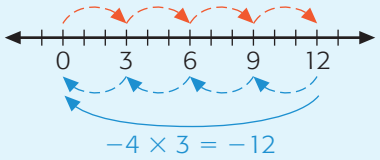
Frequently Asked Questions

Q: How can you multiply integers?

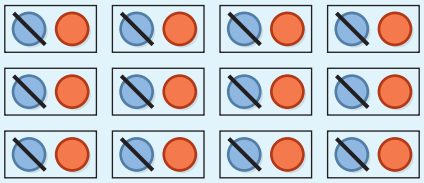
A: You can use counters, a number line, or repeated addition. The following models show that

$(+) \times (+) = +$ $\quad$ $(-) \times (+) = -$

$(+) \times (-) = -$ $\quad$ $(-) \times (-) = +$

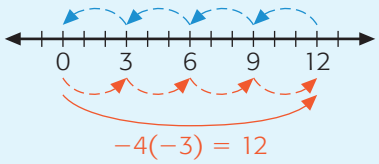
Multiplication question	Counter model	Number line model
$4 \times 3 = 12$	 4 groups of 3 positive (red) counters give a total of 12 positive (red) counters.	 The end point tells that $4 \times 3 = 12$.
$4(-3) = -12$	 4 groups of 3 negative (blue) counters give a total of 12 negative (blue) counters.	 The end point tells that $4(-3) = -12$.
$-4 \times 3 = -12$	 $-4 \times 3 = 0 - 4 \times 3$ $0 - 4 \times 3$ means subtract 4 groups of 3 red counters from 0. Use the zero principle, and add 4 groups of 3 red counters and 4 groups of 3 blue counters. Then subtract the 4 groups of 3 red counters. You are left with 4 groups of 3 blue counters, or 12 blue counters.	 The length and direction of the arrow tells that $-4 \times 3 = -12$.

$-4\ (-3) = 12$



$-4 \times (-3) = 0 - 4(-3)$

$0 - 4(-3)$ means subtract 4 groups of 3 negative (blue) counters from 0. The result is 4 groups of 3 positive (red) counters, or a total of 12 positive (red) counters.



The length and direction of the arrow tells that $-4 \times (-3) = 12$.

Practice

Lesson 6.1

1. Represent each expression using a counter model.

a) $2 \times (-5)$ **b)** $-3 \times (-4)$ **c)** -2×4

2. Calculate.

a) -6×2
b) $6 \times (-3)$
c) $-5 \times (-4)$
d) $-1 \times (-8)$
e) $0 \times (-9)$
f) $(-7)(-8)$

3. Replace the ■ with =, <, or > to make each statement true.

a) $-2 \times (-2)$ ■ -4
b) $2 \times (-10)$ ■ -20
c) $-2 \times (-5)$ ■ 7
d) $-3 \times (-1)$ ■ 4
e) $0 \times (-4)$ ■ -1
f) $3 \times (-3)$ ■ -3

Lesson 6.2

4. Represent each expression using a number line model.

a) $5 \times (-8)$ **b)** $-6 \times (-4)$ **c)** -5×9

5. When playing a game, Matt lost eight points in each of his last three turns.
Show how to use integers to determine the change in his score after these three turns.

6. Determine the least product you can form using any pair of numbers from this list. Show how you know.
$-30, -20, 0, 10, 15$

7. Create and solve a problem for $8 \times (-10)$.

6.3 Exploring Uses of Integer Division

GOAL

Investigate situations that can be modelled using integer division.

EXPLORE the Math

Elena and Vanessa keep track of changes in their weekly basketball scores. They record a positive change if a score goes up from the previous week and a negative change if a score goes down.

Elena's Basketball Record					
Week	2	3	4	5	6
Change in score	+5	−2	−2	−3	+7

Vanessa's Basketball Record					
Week	2	3	4	5	6
Change in score	−15	−8	−12	+20	+5

The girls want to compare their average weekly score changes. That means each has to add her scores and divide by the number of weeks, which requires a division involving a negative number.

? What other kinds of situations can be represented using a division involving two integers?

6.4 Integer Division

YOU WILL NEED

- red and blue counters
- Number Lines

GOAL

Use integer tiles and number lines to model integer division.

LEARN ABOUT the Math

The Science Centre uses a Van de Graaff generator to create large static charges.

Both spheres start with a neutral or 0-charge. A motor turns a belt on a pulley at a constant speed. The movement of the belt over the pulley transfers negative charges to the small sphere at a constant rate.

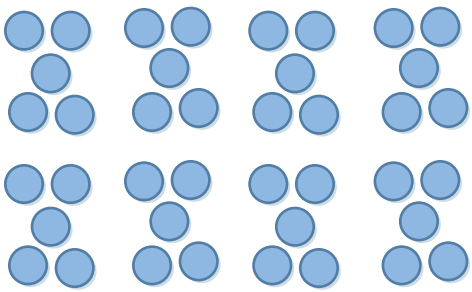
? If you know the charge and the time the motor has been running, how can you determine the charge rate?

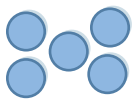
Example 1 | Dividing a 2-digit integer by a 1-digit integer

Running the motor for 8 s resulted in a charge of −40 on the small sphere and a charge of +40 on the large sphere. Determine the charge rate.

Guy's Solution: Using integer tiles to model (−) ÷ (+)

The rate at which the small sphere gained its charge is
total charge gained ÷ time = −40 ÷ 8.





$$\frac{\text{final charge of } -40}{8 \text{ seconds}} = \frac{-40}{8}$$
$$= -5$$

The small sphere gained −5 charges per second.

I knew that electrons moved to the small sphere from the larger one. That meant the small sphere gained a negative charge.

I used 40 negative (blue) tiles to represent the final charge on the small sphere.

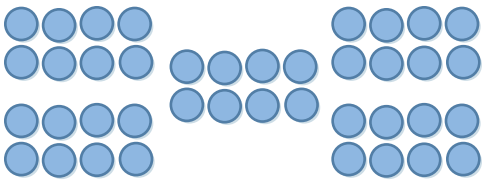
I arranged the counters into 8 equal groups, 1 group for the charge transferred each second.

The number of groups shows the number of seconds. The counters in each group represent the charge transferred each second.

The divisor represents the number of groups and the quotient represents the counters in each group.

Kaitlyn's Solution: Using integer tiles to model (−) ÷ (−)

The rate at which the large sphere lost its charge is
total charge lost ÷ time = −40 ÷ (−8).



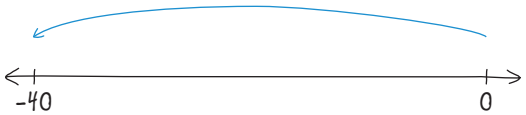
The electrons that moved to the small sphere started on the large sphere. I imagined going back in time to see how fast they had been transferred. That meant using a negative value for the 8 s.

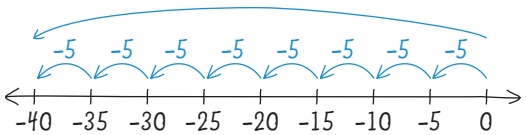
I represented dividing −40 by −8 by regrouping 40 negative (blue) counters into groups of 8. The counters in each group represent the charge transferred each second.

There are 5 groups.
The large sphere lost electrons at a rate of 5 charges/s.

The divisor represents the number of counters in each group, and the quotient represents the number of groups.

Sanjev's Solution: Using a number line to model (−) ÷ (+)





The charge rate is
final charge of $-40 \div 8$ s
$= -40$ charges $\div 8$ s
$= -5$ charges/s

I used a number line to show how the small sphere was charged.
Since the charge was −40, I drew an arrow from 0 to −40.

Then I divided the arrow into 8 equal sections, one for each second.

The number of sections is the denominator. The length and direction of each section shows the charge rate.

Mark's Solution: Relating integer division to integer multiplication

To determine the charge rate on the small sphere, I have to solve
$\square = -40 \div 8$
This is the same as solving $\square \times 8 = -40$.
$\square$ must be negative.

$\square = -5$
The small sphere was gaining charges at −5 charges/s.

I knew that every division equation has a related multiplication equation.
I knew that $5 \times 8 = 40$. If the product is negative, one of the factors has to be positive and the other has to be negative.

Reflecting

A. Why were the students correct to use division to determine the rates?

B. How can you use the signs of integers in a division to predict the sign of the quotient?

Example 2 | Solving a problem using division

The table below shows the daily low temperatures for one week in Winnipeg. Calculate the average daily low for the week.

Mon.	Tue.	Wed.	Thur.	Fri.	Sat.	Sun.
−11 °C	−11 °C	−12 °C	−10 °C	−9 °C	−9 °C	−8 °C

Solution

Total of the temperatures $= -70$.

Average daily temperature for the week

$$= \frac{-70}{7}$$
$$= -10$$

The average low for the week was −10 °C.

Calculate the average by dividing.

$$\frac{\text{total of the temperatures}}{\text{number of days}}$$

A Checking

1. Calculate.

a) $-45 \div (-5)$

b) $\frac{0}{-8}$

c) $81 \div (-9)$

d) $\frac{-56}{7}$

2. Match each division equation with the related multiplication equation. Write the missing integers.

a) $-16 \div (-8) = \square$

b) $16 \div 8 = \square$

c) $-16 \div 8 = \square$

d) $16 \div (-8) = \square$

A. $\square \times (-8) = 16$

B. $\square \times (-8) = -16$

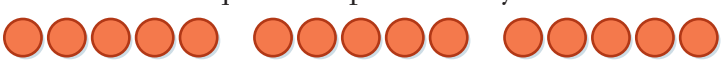
C. $\square \times 8 = 16$

D. $\square \times 8 = -16$

B Practising

3. Write the division equation represented by each model.

a)



b)

4. Write a multiplication equation for each division. Then solve the division.

a) $-72 \div (-9)$

b) $84 \div 7$

c) $66 \div (-11)$

d) $-800 \div 20$

5. Divide.

a) $40 \div (-5)$

b) $-24 \div 6$

c) $\frac{-64}{-8}$

d) $-121 \div (-11)$

e) $0 \div (-10)$

f) $\frac{54}{9}$

6. Nadia says that $\frac{-8}{-2}$ cannot represent a mean change in score. Do you agree? Why or why not?

7. Estimate each quotient.

a) $844 \div (-4)$

b) $-319 \div (-11)$

c) $448 \div (-32)$

d) $-168 \div 8$

e) $136 \div (-17)$

f) $-575 \div (-23)$

8. Determine each quotient. Multiply to check any two.

a) $\frac{48}{-12}$

b) $\frac{-32}{8}$

c) $\frac{-27}{-9}$

d) $\frac{192}{-12}$

e) $\frac{-256}{32}$

f) $\frac{-243}{-9}$

9. a) Copy and complete the following charts.

a	b	$a \times b$	Example
+	+		
+	−		
−	+		
−	−		

a	b	$a \div b$	Example
+	+		
+	−		
−	+		
−	−		

b) How is determining the sign of a product the same as determining the sign of a quotient?

10. Determine the missing integer in each equation.

a) $40 \times \square = -800$

b) $\square \times (-11) = -132$

c) $25 \times \square = 2500$

d) $\square \times 24 = -192$

11. The quotient for $-35 \div 5$ is the opposite of the quotient for $-35 \div (-5)$. Why does this make sense?

12. Explain how you know that $4 \div (-2) = -4 \div 2$ and that $(-4) \div (-2) = 4 \div 2$.

13. Emma's scores for the first nine holes of a golf game are given below.

Hole	1	2	3	4	5	6	7	8	9
Score	+1	−1	+3	+3	+2	0	0	−1	+2

Each positive integer represents a score above par. Each negative integer represents a score below par. What is Emma's mean score per hole?

14. Sanjay has a small investment. Over seven days, the value of his investment changed as shown:
$-11\ ¢, -24\ ¢, +9\ ¢, +6\ ¢, +8\ ¢, -5\ ¢, +3\ ¢$

a) What is the mean change in the value of his investments?

b) What is the difference between the mean value and the lowest recorded value?

c) What is the difference between the mean value and the highest recorded value?

15. Calculate.

a) $-3 \times (-8) \div (-4)$

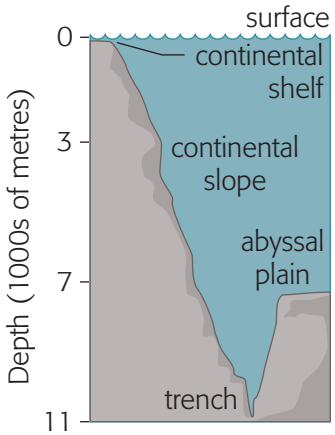
b) $\dfrac{(-6)(6)}{-4}$

c) $-63 \div (-7)(-9)$

d) $\dfrac{-144 \div 12}{-3}$

e) $(7)(-6) \div (3)(-7)$

f) $(-2)(-9) \div (2)(-3)$



16. The Marianas Trench is the deepest spot in the world's oceans. It is located in the Pacific Ocean, just east of the Philippines. The maximum depth of the Marianas Trench is 10 962 m. The maximum depth of Lake Superior is 406 m.
Create and solve an integer division question using this information.

17. Predict the sign for each quotient without actually computing the final answer. Explain how you know.

a) $\dfrac{-125}{25}$

b) $\dfrac{84}{-7}$

c) $\dfrac{-91}{-7}$

6.5 Order of Operations

YOU WILL NEED
- a calculator

GOAL

Apply the rules for the order of operations with integers.

LEARN ABOUT the Math

Joseph won a contest, but he has to answer the following skill-testing question before he can claim the prize:

$$\frac{6 \div (-3) + [(4 - (-5)) \times (-7)]}{4 - 5}$$

Communication | *Tip*

Rules for Order of Operations

- Evaluate the contents of brackets first.
  If there are brackets within brackets, perform the operations in the innermost brackets first.
- Treat the numerator and denominator of a fraction as if they were each in brackets.
- Divide and multiply from left to right.
- Add and subtract from left to right.

What is the answer to the skill-testing question?

Example 1 | Evaluating an expression in fraction form

Use the order of operations to evaluate the skill-testing question.

Joseph's Solution

$$\frac{6 \div (-3) + [(4 - (-5)) \times (-7)]}{4 - 5}$$

I used the same order of operations for integers as I would have for other numbers.
I started by determining the value of the numerator.
I calculated what is in the innermost brackets.

$$= \frac{6 \div (-3) + [9 \times (-7)]}{4 - 5}$$

I calculated what is in the square brackets.

$$= \frac{6 \div (-3) + [-63]}{4 - 5}$$

I divided.

$$= \frac{-2 + [-63]}{4 - 5}$$

I added to calculate the numerator.

$$= \frac{-65}{4 - 5}$$

I subtracted to calculate the denominator.

$$= \frac{-65}{-1}$$

I divided the numerator by the denominator.

$$= 65$$

The answer is 65.

Reflecting

A. Chiyo says, "If an expression has a numerator and a denominator, like $\frac{-6 + (-10)}{(-4)(2)}$, the last calculation is division." Is Chiyo correct? Explain.

B. If you used a different order of operations, would your answer be different? Explain. Use an example.

WORK WITH the Math

Example 2 | Calculating using fewer steps

Calculate the answer to this skill-testing question.

$$\frac{[8 \times (-2) - 7] - 10 \div (-5)}{-3 - 4}$$

Vanessa's Solution

Numerator
[8 x (-2) - 7] - 10 ÷ (-5)

I calculated the numerator first. I did three steps at the same time, since the calculations do not affect each other.
$8 \times (-2) = -16$, $-16 - 7 = -23$, and $10 \div (-5) = -2$.

-23 - (-2) = -21

The numerator is -21.

Denominator
-3 - 4 = -7

Then I calculated the denominator. It is like an expression in brackets.

$\frac{-21}{-7} = 3$

I know that a fraction can represent division. I also know that a negative integer divided by a negative integer is positive.

A Checking

1. Calculate.

a) $-9 - (-6) \div 6$

b) $4 \times (-8) - (-5)$

c) $-8 \times (-3) - (-8) \div (-4)$

d) $\dfrac{-16}{[-2 - (-18)] \times (-1)}$

B Practising

2. In each expression, which calculation(s) should you do first?

a) $-5 + (-6) \times (-8) \div 2$

b) $-8 \times 6 \div (-2) - [-9 \times (-3)]$

3. Calculate.

a) $-2 + (-3) \times (-8 + 4)$

b) $-9 - (-8) \times 7 + [6 \times (-2)]$

c) $7 \times [8 - (-2) \times (-6)]$

d) $[-2 - (-8)] \times (-5)$

e) $35 + (-4) \times (-8) - 7$

f) $18 \times (-3 - [8 \times (-5)])$

4. There is an error in this solution.

$$\begin{aligned} 3 \times (-8) \div (-2 - 4) &= -24 \div (-2 - 4) \\ &= 12 - 4 \\ &= 8 \end{aligned}$$

a) Find the error.

b) Explain how to correct the error.

5. Calculate.

a) $\dfrac{-6 + (-10)}{(-4)(2)}$

b) $\dfrac{49 \div (-7)}{1 + (-2)(-3)}$

c) $\dfrac{28 \div (-4 - 3)}{(-2 + 4) \times 2}$

d) $\dfrac{27 + (-18) \div (-2)}{(-2 + 5)(2)}$

e) $\dfrac{-9 + (-16) - 10}{(-7)(10) \div (-2)}$

f) $\dfrac{[6 + (-38)] \div 4(-2)}{(-2 + 4)(5 - 6)}$

6. **a)** Evaluate with a calculator.

$-147 + 156 \div (-4) + 405 \div (-15)$

b) Does your calculator follow the order of operations? How do you know?

7. Using brackets, group the terms in this expression to get the least possible result.

$40 \times 6 - 3 \times 4 - 5$

8. The formula for converting temperatures from Fahrenheit (F) to Celsius (C) is $C = (F - 32) \times 5 \div 9$.
Use the formula to calculate -40 °F in degrees Celsius.

9. This chart shows the predicted high temperatures in Iqaluit for a week in November. Use an integer expression to determine the mean predicted high temperature for the week.

Day	Predicted high temperature (°C)
Monday	−4
Tuesday	−4
Wednesday	0
Thursday	1
Friday	−1
Saturday	−2
Sunday	−4

10. Copy each equation. Identify the missing operation signs.

a) $36 \square (4 \square 1) \square 2 = 24$

b) $-12 \square 4 \square (-3) = -24$

c) $-15 \square (-12) \square 6 \square 16 = -47$

11. Adrian bought some shares in four companies. This chart shows how his shares changed in value over one month. Write an integer expression that could be used to determine the change in the total value of his shares. Evaluate your expression.

Company	A	B	C	D
Number of shares	10	100	50	30
Value of each share Aug. 1 ($)	42	5	38	19
Value of each share Sept. 1 ($)	39	4	42	21

12. The price of gold changes daily. One week, the price started at \$675 per ounce on Monday and changed −\$2 each day for 3 days, and then +\$8 each day for the next 2 days.

a) Complete the chart.

Day	Starting price ($)	Final price ($)	Change in price ($)
Monday	675		
Tuesday			
Wednesday			
Thursday			
Friday			

b) Calculate the mean final price of gold for the week.

c) Calculate the mean change in price for the week.

13. Create an integer expression that shows why the rules for the order of operations are needed. Explain how your expression shows this.

14. How is the process for calculating the value of an integer expression the same as the one you use for a whole number expression? How is it different?

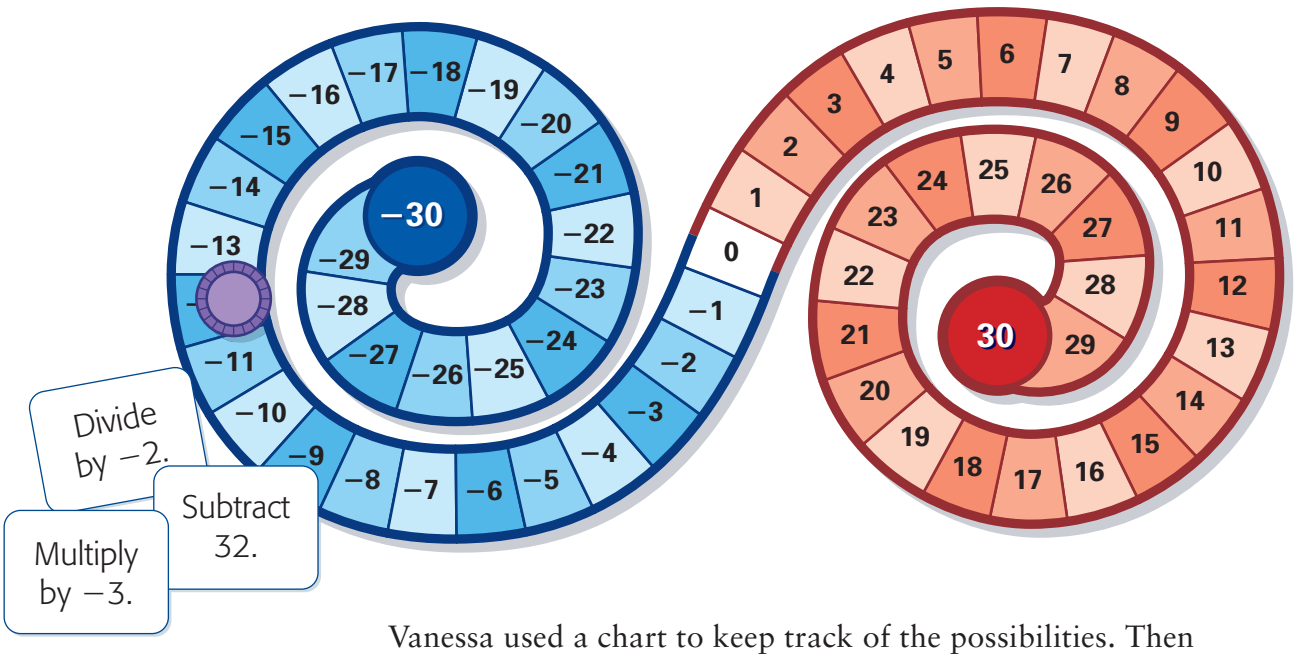
6.6 Communicate about Problem Solutions

GOAL

Explain the process of solving an integer problem.

LEARN ABOUT the Math

The cards below were the last three cards that Guy was dealt in a game. Guy followed the instructions correctly, but maybe not in the order shown. Now he is at −12 on the board. Where was he three turns ago?



Vanessa used a chart to keep track of the possibilities. Then she asked Elena to read her solution. Elena asked Vanessa the questions shown.

Vanessa's Solution

If the last card was ...	Guy came from ...	If the card before was ...	Guy came from ...	If the other card was ...	Guy came from ...
Divide by -2.	24	Subtract 32.	56: impossible		
Divide by -2.	24	Multiply by -3.	-8	Subtract 32.	24
Subtract 32.	20	Multiply by -3.	impossible (not an integer)		
Subtract 32.	20	Divide by -2.	-40: impossible		
Multiply by -3.	4	Divide by -2.	-8	Subtract 32.	24
Multiply by -3.	4	Subtract 32.	36: impossible		

Elena's Questions

Why did you use a chart to solve the problem?

How did you choose the column headings?

How did you fill each "Guy came from ..." column?

How did you fill each "If the ... was ..." column?

Did you justify your conclusion that a position was not possible?

Did you state and justify your result for the problem?

Communication *Checklist*

- ✔ Did you identify the information given?
- ✔ Did you show each step in your solution?
- ✔ Did you explain your thinking at each step?
- ✔ Did you check that your answer is reasonable?
- ✔ Did you state your conclusion clearly?

How can Vanessa improve her solution?

A. Which of Elena's questions do you think are good questions? Why?

B. How should Vanessa answer Elena's questions?

C. What other questions would be helpful to improve Vanessa's work?

Reflecting

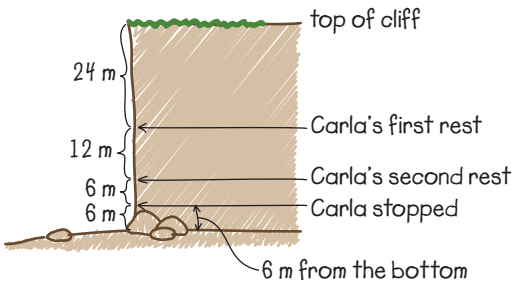
D. Which parts of the Communication Checklist did Elena cover well?

WORK WITH the Math

Example 1 | Using a diagram to explain a solution

Carla climbed halfway down a cliff before resting the first time. Then she climbed halfway down the remaining distance and rested for a second time. After climbing halfway down the final distance, Carla was 6 m from the bottom of the cliff. Use an integer to describe Carla's distance, in metres, from the top of the cliff. Explain your thinking.

Joseph's Solution



I drew a diagram to show the stages in Carla's climb down the cliff. I imagined that the top of the cliff was 0, and that down was negative and up was positive.
Carla finished 6 m from the bottom of the cliff, halfway between the bottom of the cliff and the location of her second rest. So she climbed down 6 m after her second rest. That is $0 + (-6)$.

At her second rest, Carla must have been 12 m from the bottom.

Carla's second rest was halfway between the bottom of the cliff and the location of her first rest. At her first rest, Carla must have been 24 m from the bottom. So she climbed down 12 m after her first rest. That is $0 + (-6) + (-12)$.

Carla's first rest was halfway between the bottom and the top of the cliff. So she climbed down 24 m before her rest. That is -24 m.

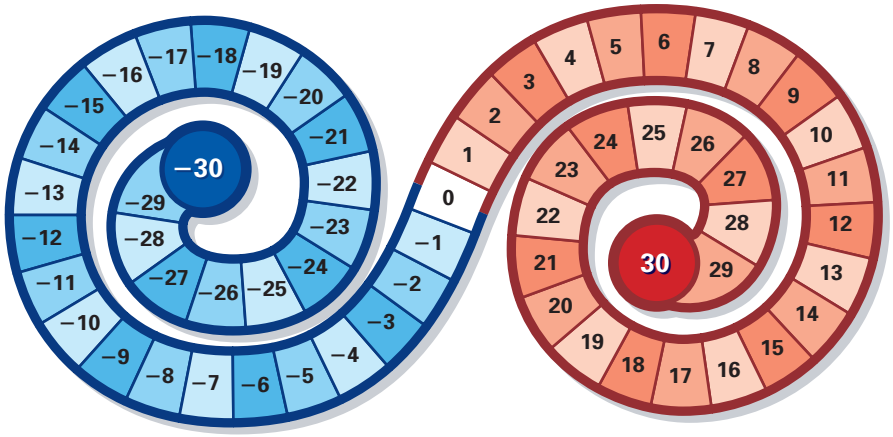
That is $0 + (-6) + (-12) + (-24)$.

$0 + (-6) + (-12) + (-24) = -42$

The integer -42 describes Carla's distance in metres from the top of the cliff.

A Checking

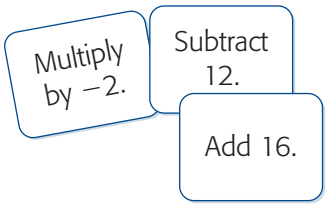
Use the game board below to answer questions 1 to 3.



1. a) Write the instructions for three cards to go from −10 to 10 on the game board. Then write two other solutions.

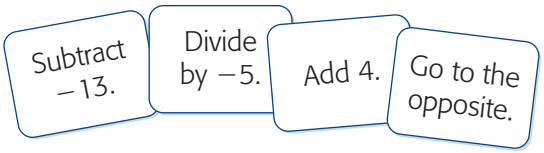
b) Rewrite the instructions in part a) to go from 10 to −10. Explain your thinking.

B Practising



2. These were Guy's last three cards before he landed on −2. Where did he begin? How do you know?

3. These were Guy's last four cards before he landed on −7. Where could he have begun? Explain how you determined your answers.



4. Samara walked 3 km to the west. Then she walked twice as far going toward the east. She continued toward the east for another kilometre, stopping 2 km east of Lauren's home. When Samara started walking, how far was she from Lauren's home? Explain how you know.

5. a) Change a problem in this lesson to create a different problem, or make up a new integer problem.

b) Explain how to solve the problem.

Peasant Multiplication

Russian peasants used this method to multiply whole numbers without the use of multiplication tables.

The Peasant Multiplication Algorithm

A. 11×23

11	23
B. 5	46
~~2~~	~~92~~
1	184

C. $23 + 46 + 184 = 253$
$11 \times 23 = 253$

Write the two numbers you wish to multiply in two columns.
Divide the number on the left by two. Ignore any fractional portion. Double the number on the right.
If the number on the left is even, cross out the entire line.
Repeat the steps until the number on the left is 1.
Add the numbers in the right column that have not been crossed out.

1. Use Peasant Multiplication to calculate each product.

a) 47×15
b) 15×47
c) 36×47
d) 47×36

2. Why is it a good idea to place the lower multiplier in the left column?

3. Use Peasant Multiplication to help you calculate each integer product.

a) -25×13
b) $43(-17)$
c) $-16(-21)$
d) $-23\,958 \times 584$

Math **GAME**

Target Zero

Number of players: 2 to 4

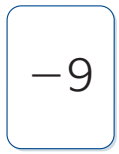
When using a standard deck of cards, aces count as 1, numbered cards count as their face values, and jokers count as 0. Red cards are positive, and black cards are negative.

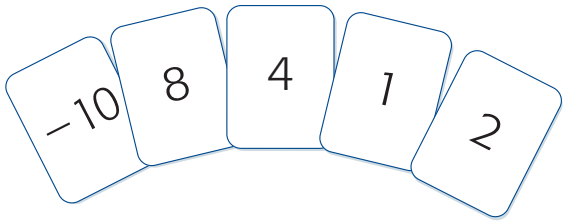
YOU WILL NEED

- Integer Cards (two of each card)
  OR
  standard deck of cards (including 2 jokers) with face cards removed

Rules

1. Shuffle the cards. Deal five cards to each player.
2. Place the remaining cards in a pile with one card facing up. This is the target card.





3. Players have 1 min to write an integer expression that uses all of their five cards and has a value as close as possible to the value of the target card. The integers can be combined using operations and brackets.

$(-10 + 8)(4 - 1 + 2)$

$-10 + 8 + 4 - 1 - 2$

4. Players evaluate their expressions.

   Each player receives a score equal to the positive difference between the value of her or his expression and the value of the target card. An exact match gives a score of 0.

$(-10 + 8)(4 - 1 + 2) = -10 \rightarrow$ score is 1

$-10 + 8 + 4 - 1 - 2 = -1 \rightarrow$ score is 8

5. Repeat steps 1 to 4 ten times. The winner is the player with the lowest final score.

Chapter Self-Test

1. Use counters or a number line to represent each expression.

a) $5 \times (-2)$

b) $-2 \times (8)$

c) $6 \times (-10)$

d) $-5 \times (-5)$

e) $\frac{-25}{5}$

f) $-36 \div (-9)$

2. Calculate.

a) $6 \times (-1)$

b) -9×3

c) $-12 \times (-12)$

d) $-96 \div (-16)$

e) $-98 \div 14$

f) $88 \div (-11)$

3. Determine the missing values.

a) $-34 \times \square = 306$

b) $28 \times \square = -336$

c) $\square \div 8 = -7$

d) $\square \div (-18) = 23$

4. Marcus recorded this information about his shares. Estimate how much money, in total, he has gained or lost.

Company	A	B	C	D
Number of shares	50	70	100	25
Change in price per share ($)	−2	+5	−3	−8

5. Which two integers have a product of -120 and a sum of -2?

6. How much greater or less is $5 + (-2)(-8)$ than $-8 + (-2)(5)$?

7. Calculate.

a) $-2 \times (-5)$

b) $-12 \div 4 \times (-7) - (-2) + 10$

c) $18 \times [-3 - (8)(-5)]$

d) $\frac{-54 + 18 \div (-2)}{(-3 - 4)(-1)}$

8. Use the following integers once each, and any necessary arithmetic operations and brackets, to make an expression equal to -96.

$-10, -4, 3, 4, 10$

What Do You Think Now?

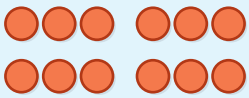
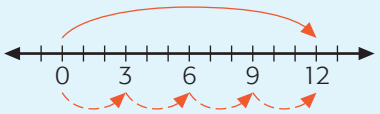
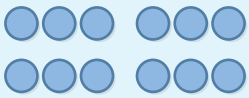
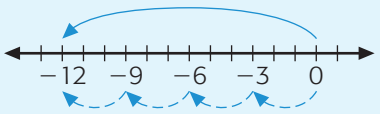
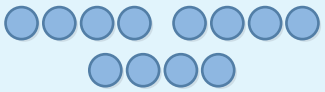
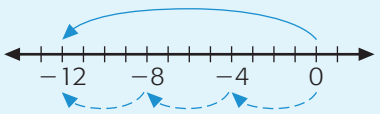
Revisit What Do You Think? on page 243. How have your answers and explanations changed?

Chapter Review

Frequently Asked Questions

Q: How can you divide integers?

A: You can use counters, a number line, or a related multiplication equation.

Division question	Counter model	Number line model	Related multiplication
$12 \div 3 = 4$	 The number of groups is the quotient.	 The number of small arrows is the quotient.	$12 \div 3 =$ ▢ is related to ▢ $\times 3 = 12$
$-12 \div (-3) = 4$	 The number of groups is the quotient.	 The number of small arrows is the quotient.	$-12 \div (-3) =$ ▢ is related to ▢ $\times (-3) = -12$
$-12 \div 3 = -4$	 The number of blue counters in each group is the quotient.	 The length and direction of each small arrow is the quotient.	$-12 \div 3 =$ ▢ is related to ▢ $\times 3 = -12$
$12 \div (-3) = -4$	Dividing a positive integer by a negative integer cannot be represented easily using counters or a number line.		$12 \div (-3) =$ ▢ is related to ▢ $\times (-3) = 12$

Q: How do you evaluate integer expressions that involve several operations?

A: Follow the same order of operations that you use with whole numbers and decimals:

Evaluate the contents of brackets first. Divide and multiply from left to right. Add and subtract from left to right.

Practice

Lesson 6.1

1. Calculate.

a) -4×4

b) $-8 \times (-2)$

c) $0 \times (-1)$

d) -5×4

2. Predict the sign of each product without calculating it. Explain how you predicted.

a) $-1 \times (-2) \times 4$

b) $4 \times (-2) \times 5$

Lesson 6.2

3. Calculate.

a) $-10(-8)$

b) $16(-5)$

c) $-6(12)$

d) $-21(-11)$

4. The product of five different integers is -24. Write two possible lists of integers for which this is true.

Lesson 6.4

5. Calculate.

a) $-32 \div 8$

b) $36 \div (-9)$

c) $-27 \div (-3)$

d) $75 \div (-25)$

6. Determine the mean of each group of integers.

a) $10, -8, -16, -6$

b) $-21, 9, 15, -30, -3$

Lesson 6.5

7. Replace each ■ in the equations with $+$, $-$, $\times$, or $\div$.

a) $-58 \blacksquare (-36) \blacksquare (-15) = -37$

b) $-4 \blacksquare (-3) \blacksquare 28 = 40$

8. Estimate.

a) $9 \times (-3) + (-15) \div 3$

b) $(-45) \div 5 + 7 - (-12)$

9. Melissa says, "When I combine integers using several operations, I always get the right answer if I do the operations from left to right." Use examples to explain whether she is right or wrong.

Chapter Task

Task | *Checklist*

- ✔ Did your clues involve all four operations with integers?
- ✔ Did at least one of your clues require comparing integers?
- ✔ Did you use appropriate math language?
- ✔ Did you check to see that your clues worked?

Mystery Integers

Select four integers. Do not tell anyone what they are. Make up a set of four clues that will allow someone to guess the integers you chose. All four of the clues must be necessary.

The clues must

- use all four operations somewhere in the four clues
- include comparing integers

For example, suppose that your integers are -8, 7, 5, and -3.

Here are three possible clues:

- The sum of the four integers is 1.
- If you order the integers from least to greatest, the product of the two middle integers is -15.
- If you subtract the least integer from the greatest integer, and divide the difference by 3, the quotient is 5.

What four clues can you write to describe your four integers?

A. The three clues above do not give enough information to figure out the integers. What additional clue would give enough information?

B. Select any four integers of your own and make up four clues. Remember that all of the clues must be necessary. It should not be possible to figure out all the integers with only some of the clues.