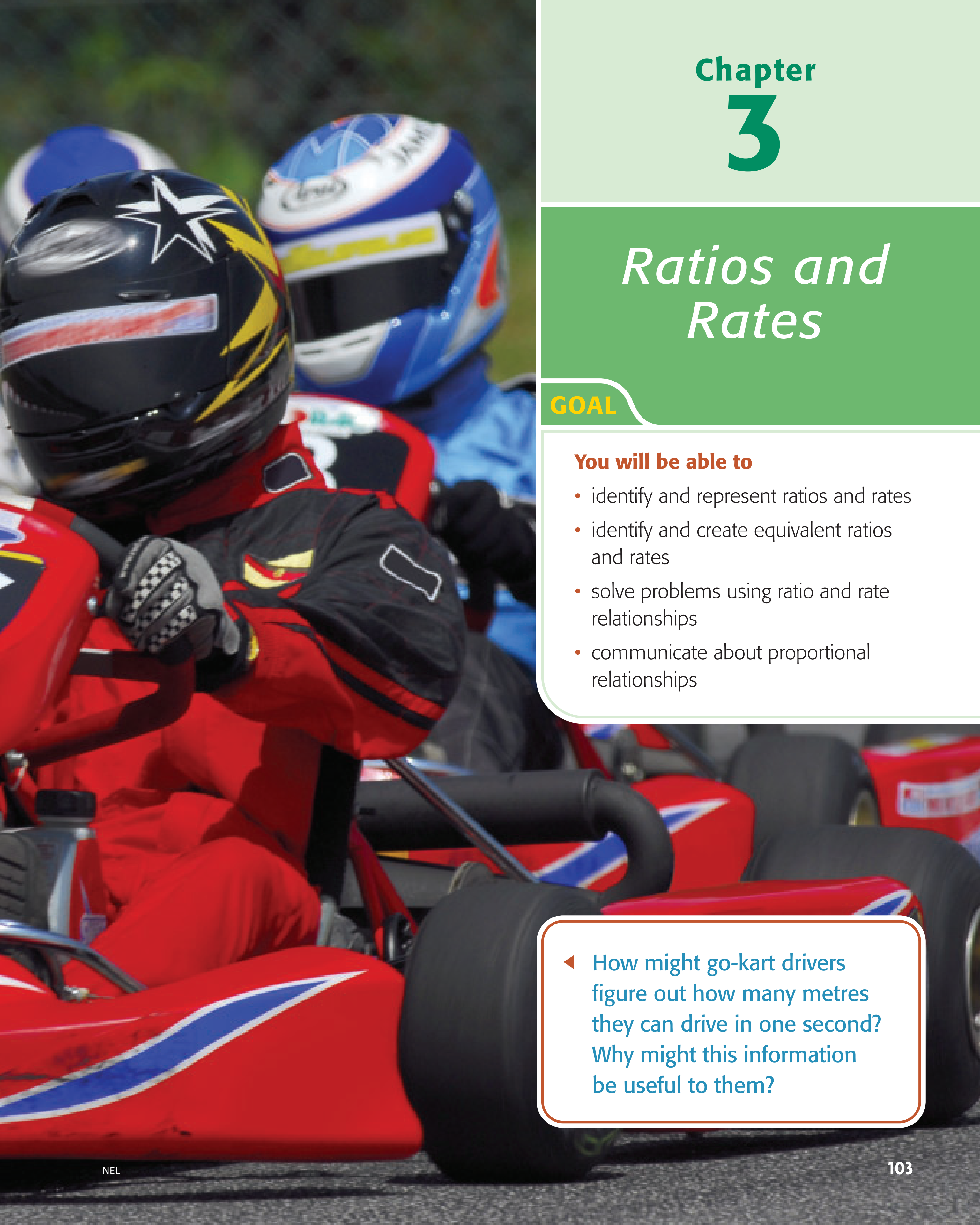


Chapter
3

Ratios and Rates

GOAL

You will be able to

- identify and represent ratios and rates
- identify and create equivalent ratios and rates
- solve problems using ratio and rate relationships
- communicate about proportional relationships

◄ How might go-kart drivers figure out how many metres they can drive in one second? Why might this information be useful to them?

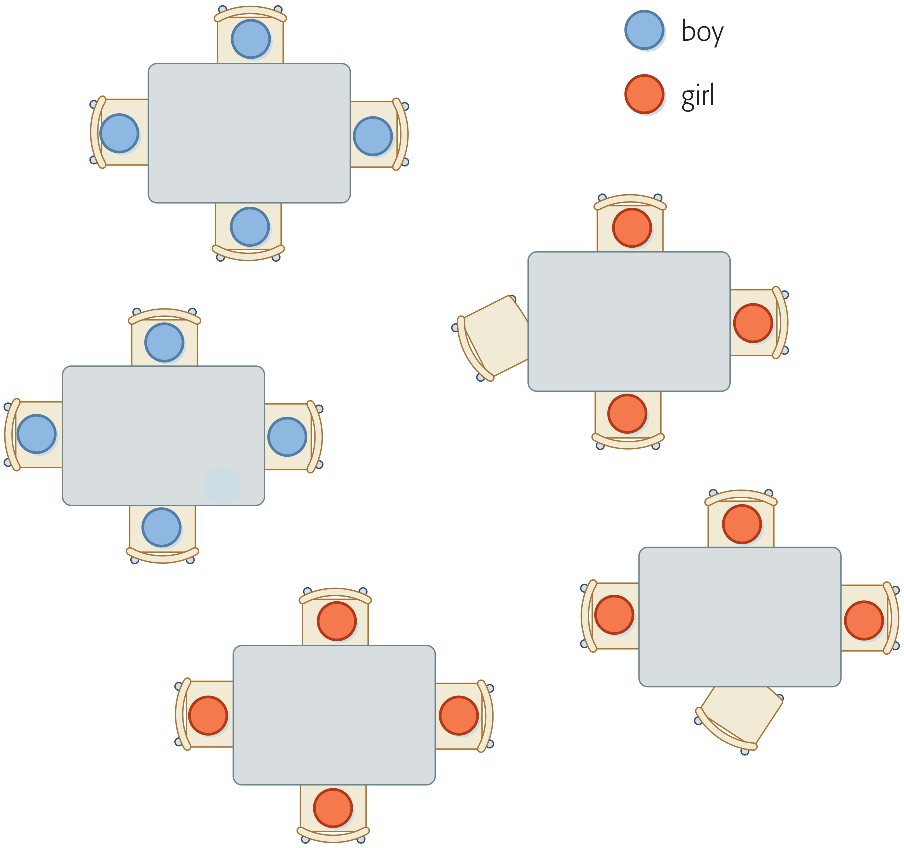
Getting Started

YOU WILL NEED

- red and blue counters

Seating Arrangements

Ten girls and eight boys are sitting in the cafeteria as shown.



What ratios could describe their seating arrangement?

A. Explain how the **ratio** 10 : 8 describes the students.

ratio

a comparison of two numbers (e.g., 5 : 26 is the ratio of vowels to letters in the alphabet) or of two measurements with the same units (e.g., 164 : 175 is the ratio of two students' heights in centimetres). Each number in the ratio is called a term.

part-to-part ratio

a comparison of two parts of the same whole

🟥🟥🟦🟦🟦🟦

(e.g., 2 : 4 compares the number of red tiles to the number of blue tiles)

part-to-whole ratio

a comparison of part of a whole to the whole

🟥🟥🟦🟦🟦🟦

(e.g., 2 : 6 compares the number of red tiles to the total number of tiles) that can be written as a fraction, such as $\frac{2}{6}$

B. Write another **part-to-part ratio** to describe the students.

C. Write a part-to-part ratio to compare the number of tables with boys to the number of tables with girls.

D. Write a **part-to-whole ratio** to compare the number of tables with boys to all the tables. Write the ratio as a fraction.

E. Write a part-to-whole ratio to describe the number of tables with girls to all the tables. Write the ratio in the form “□ to □”.

F. Suppose three boys and one girl sat at one table. What would each of these ratios describe?

- 3 : 1
- 1 to 4
- $\frac{3}{4}$

G. Suppose the ratio 2 : 2 represents the students at a table. Who might be sitting at the table?

H. Draw five squares to represent the five tables.

- Arrange 10 red and 8 blue counters to represent the girls and boys at the tables.
- Sketch your model.
- List all the different ratios your diagram shows.
- Explain how each ratio represents your seating arrangement.

What Do You Think?

Decide whether you agree or disagree with each statement. Be ready to explain your decision.

1. The first term of a ratio should always be less than the second term.

2. The ratios 2 : 2 and 3 : 3 describe the same comparison.

3. If you know a part-to-part ratio, you can always calculate the related part-to-whole ratio.

4. Prices are like ratios since they compare two numbers.

3.1 Using Two-Term Ratios

GOAL

Compare two quantities using ratios.

LEARN ABOUT the Math

Nikita used a Ukrainian recipe for cold fruit soup, but to make more, she used 6 cups of cranberry juice.

Cold Fruit Soup

Liquids

4 cups cranberry juice

3 cups white grape juice

8 cups water

Solids

2 cups sugar

1 cup raspberry jam

How much water and grape juice should she use?

A. Write the part-to-part ratio of cranberry juice to water.

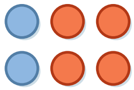
B. Draw a picture to show why 2 : 4 is an **equivalent ratio** to the ratio in part A.

C. Write a **proportion** to determine the amount of water needed for 6 cups of cranberry juice.

D. Write the part-to-part ratio of cranberry juice to grape juice.

equivalent ratio

a ratio that represents the same relationship as another ratio; e.g., 2 : 4 is an equivalent ratio to 1 : 2 because both ratios describe the relationship of the blue counters to the red counters. There are 2 red counters for each blue counter, but also 4 red counters for every 2 blue counters.



proportion

a number sentence that shows two equivalent ratios or fractions; for example, $1:2 = 2:4$ or $\frac{1}{2} = \frac{2}{4}$

E. Write equivalent ratios you can use to determine how much grape juice is needed in each case.
- You use 2 cups of cranberry juice.
- You use 6 cups of cranberry juice.

F. How much water and grape juice should Nikita use?

Reflecting

G. How are the three ratios in parts A, B, and C related?

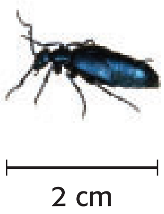
H. How is creating an equivalent ratio like creating an equivalent fraction?

WORK WITH the Math

Example 1 | Representing situations with ratios

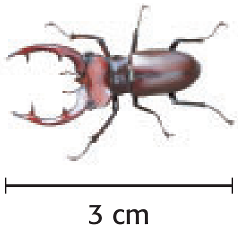
Compare the lengths of the blister beetle and the hydraena beetle to the length of the giant stag beetle using ratios.

blister beetle



hydraena beetle

giant stag beetle



Brian's Solution

	I think the ratios should be different since the beetles are such different sizes.
blister beetle and giant stag beetle: 2 cm : 3 cm The ratio is 2 : 3.	The ratio 2 : 3 makes sense since the stag beetle is $1\frac{1}{2}$ times as long as the blister beetle, just like 3 is $1\frac{1}{2}$ times 2.
hydraena and giant stag : 3 cm = 30 mm	I renamed 3 cm as 30 mm. That way, I was comparing 2 mm to 30 mm.
2 mm : 30 mm The ratio is 2 : 30.	That seems right, since $30 = 15 \times 2$ and the stag beetle looks like it is 15 times as long as the hydraena beetle.

Example 2 | Creating equivalent ratios using fractions

Misa has chat room buddies from Canada and the U.S.A. in a ratio of 30 : 20.
She has 135 Canadian buddies. How many U.S. buddies does she have?

Allison's Solution

$30:20 = \frac{30}{20}$	I wrote the ratio as a fraction.
$\underset{\div 10}{\overset{\div 10}{\frac{30}{20} = \frac{3}{2}}}$	I renamed $\frac{30}{20}$ in lower terms by dividing the numerator and denominator by the common factor of 10.
$\frac{\text{Canadian buddies}}{\text{U.S. buddies}} = \frac{3}{2}$ $= \frac{135}{\square}$	I wrote a proportion comparing Canadian buddies to U.S. buddies.
$135 \div 3 = 45$ $\underset{\times 45}{\overset{\times 45}{\frac{3}{2} = \frac{135}{90}}}$	I divided 135 by 3 to figure out what to multiply 2 by.

Misa has 90 U.S. buddies.

Example 3 Solving a ratio problem using a proportion

The ratio of the mass of oats to barley in some horse feed is 4 : 11.
How many kilograms of each grain are in 150 kg of feed?

Aaron's Solution

$\underbrace{O\,O\,O\,O}_{4 \text{ parts}}\underbrace{B\,B\,B\,B\,B\,B\,B\,B\,B\,B\,B}_{11 \text{ parts}}$	I represented the ratio with a diagram. I represented oats with O and barley with B.
The whole is 15.	4 : 11 is a part-to-part ratio. I added the parts to determine the whole.
Oats = $\frac{4}{15}$ of total mass Barley = $\frac{11}{15}$ of total mass	I wrote fractions to describe what part of 15 kg of feed is oats and what part is barley.
Oats: $\frac{4}{15} \underset{\times 10}{=} \frac{\square}{150}$	To find out the amount of oats in 150 kg of feed, I needed an equivalent fraction for $\frac{4}{15}$ with a denominator of 150.
$\frac{4}{15} \overset{\times 10}{\underset{\times 10}{=}} \frac{40}{150}$ Mass of oats = 40 kg	I multiplied the denominator by 10, so I had to multiply the numerator by 10.
Mass of barley = 150 – 40 = 110 In 150 kg of feed, there are 40 kg of oats and 110 kg of barley.	I subtracted to figure out how much of the mass is barley.

Misa's Solution

4 + 11 = 15	I figured that if there were 4 kg of oats and 11 kg of barley, there would be 15 kg of feed.
150 ÷ 15 = 10 4 x 10 = 40 kg oats 11 x 10 = 110 kg barley	I realized that 150 is 10 times as much as 15, so the parts would also have to be 10 times as much.

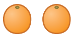
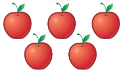
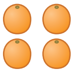
A Checking

1. a) Write a part-to-part ratio to compare the items in the top row in each diagram.

b) The bottom row represents an equivalent ratio. Write a proportion you could solve to calculate the missing term.

c) Calculate the missing term.

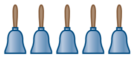
A.

2 oranges 	5 apples 
4 oranges 	■ apples

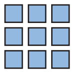
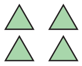
B.

3 women 	5 men 
■ women	15 men 

C.

2 stars 	5 bells 
8 stars 	■ bells

D.

3 triangles 	9 squares 
4 triangles 	■ squares

2. Calculate each missing term.

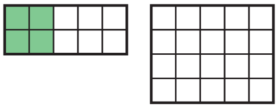
a) $3:8 = ■:16$

b) $20:■ = 32:24$

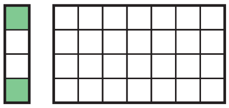
B Practising

3. Copy the grid on the right. Shade it so the ratio of coloured squares to the total number of squares is the same as for the grid on the left.

a)



b)



4. Write three equivalent ratios for each ratio.

a) 21 to 56

b) $6:54$

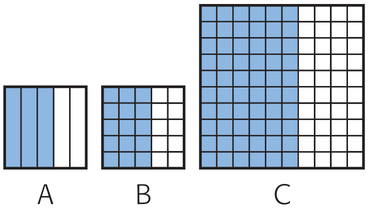
c) $48:36$

d) $\frac{22}{55}$

e) 7 to 42

f) $\frac{3}{8}$

5. Show that the ratio of the number of blue sections to the total number of sections is the same for all three diagrams.



6. Determine the missing term in each proportion.

a) $27:45 = \square:5$

b) $\frac{2}{6} = \frac{3}{\square}$

c) $\square:9 = 2:3$

d) $\frac{16}{\square} = \frac{12}{15}$

7. Write each comparison as a ratio. Remember, the units should be the same so the comparison is meaningful.

a) 400 g to 1 kg **b)** 6 cm to 7 mm **c)** 200 s to 3 min

8. **a)** Draw one picture to show the ratios $3:1$ and $\frac{3}{4}$ at the same time.

b) Explain how each ratio is in the picture.

c) Change your picture to show two equivalent ratios to the ones in part a).

Reading Strategy

Activating Prior Knowledge

What do you know about fractions? How can what you know about fractions help you with ratios?

9. Katherine spends 7 h each day in school, including 30 min for lunch.

a) Write a ratio to compare the time for lunch with the total time in school each day.

b) Write a ratio to compare the total time for lunch for a school week with the total school time for a school week.

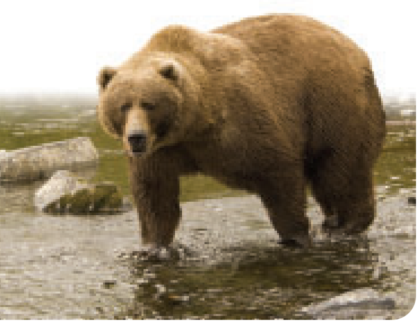
c) Calculate the number of hours of lunch time in 30 school days.

10. Suppose you add the same amount to both terms of a ratio. Which of the following is true? Explain.

A. You never get an equivalent ratio.

B. You usually do not get an equivalent ratio.

C. You usually get an equivalent ratio.



11. Park rangers captured, tagged and released 82 grizzly bears. A month later, the rangers captured 20 bears, two of which had tags. Estimate the park's grizzly bear population.

12. Lucas is 9 years old and 129 cm tall. Medical charts show that a boy's height at age 9 is $\frac{3}{4}$ of his predicted adult height. Predict Lucas's adult height.

13. Using a ratio, compare two related quantities or measurements you find in a newspaper article, on the Internet, or on TV. Describe the ratio and explain why it is actually a ratio.

14. George rolled a die 30 times, with these results.

Roll of	⚀	⚁	⚂	⚃	⚄	⚅
How many times	4	5	6	5	8	2

a) Explain how $\frac{8}{30}$ describes the experimental probability of rolling a 5. What sort of ratio is it?

b) Write a part-to-part ratio to compare the rolls of 5 to the rolls of 1.

c) Are any other part-to-part ratios equivalent to the ratio in part b)?

d) Suppose George were to roll a die 10 times instead of 30. About how many rolls would you expect for each number? Explain.

15. Describe a situation you might represent with each ratio.

a) $\frac{3}{60}$

b) 9 : 20

16. The sides of a rectangle with an area of 192 cm^2 are in the ratio 3 : 4. What are the side lengths?

17. A ratio can be called a multiplicative comparison of two amounts. Why is a ratio about multiplication?

3.2 Using Ratio Tables

GOAL

Use ratio tables to solve problems.

LEARN ABOUT the Math

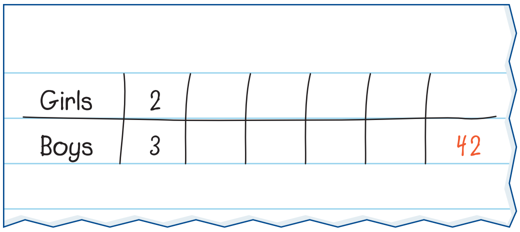
The students in Preston's school signed up for a snowboard trip.
Two girls signed up for every three boys who signed up.
In all, 42 boys signed up.

? How many girls signed up for the trip?

Example 1 | Using a ratio table

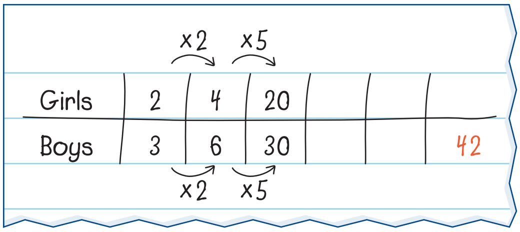
I used a ratio table to help me solve the problem.

Preston's Solution



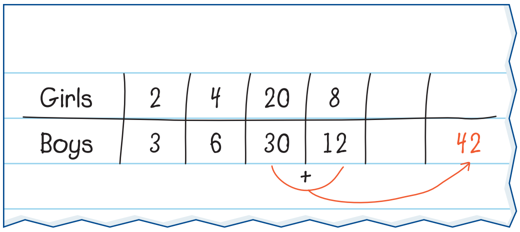
Girls	2					
Boys	3					42

The ratio of girls to boys is 2 : 3.
I drew a two-row table and wrote the ratio of girls to boys in the first column. I decided to put girls on the top row, but I did not have to. I needed an equivalent ratio for 42 boys.



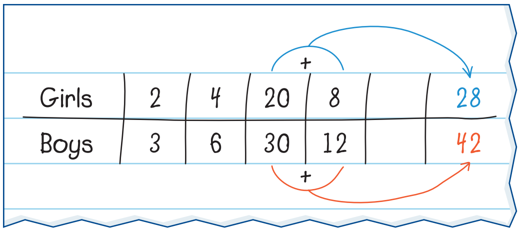
Girls	2	4	20			
Boys	3	6	30			42

I created some equivalent ratios, hoping I would see a connection to a ratio with 42 boys. I got equivalent ratios by multiplying both numbers in a column by the same amount. I picked numbers that were easy to multiply by. I wrote the equivalent ratios in other columns.



Girls	2	4	20	8		
Boys	3	6	30	12		42

I saw that I had a column with 30 boys, and I knew I needed to have a column with 42 boys in it. I realized that, if I multiplied the second column by 2, I would get a column with 12 boys. Then I could add the columns with 30 boys and 12 boys to get a column with 42 boys.



Girls	2	4	20	8		28
Boys	3	6	30	12		42

So 28 girls signed up for the trip.

The ratio 28 : 42 is equivalent to the original ratio of 2 : 3.

Reflecting

A. Why do you get an equivalent ratio when you multiply the numbers in one column by the same amount?

B. Why did Preston get an equivalent ratio when he added the column with 20 and 30 and the column with 8 and 12?

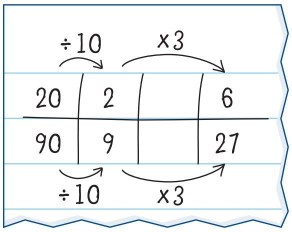
C. What other ratio table could you have used to solve the same problem?

WORK WITH *the Math*

Example 2 | Solving a proportion using a ratio table

Solve $\frac{6}{\square} = \frac{20}{90}$ using a ratio table.

Allison's Solution



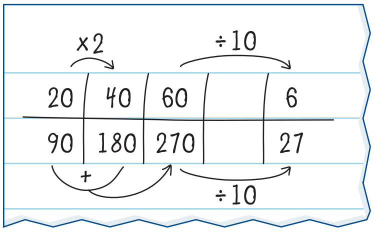
20	2		6
90	9		27

$\frac{6}{27} = \frac{20}{90}$

The missing term is 27.

I started the table with the ratio I knew, $\frac{20}{90}$.
I wanted a first term of 6.
I noticed that 20 and 90 had a common factor of 10, so I divided them by 10 to get an equivalent ratio in lower terms.
Then I multiplied 2 by 3 to get 6.
I multiplied 9 by 3, since both terms must be multiplied by the same number.

Brian's Solution



20	40	60		6
90	180	270		27

$\frac{6}{27} = \frac{20}{90}$

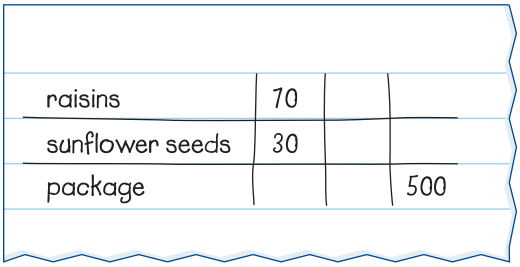
The missing term is 27.

I started the table with the ratio in which I knew both terms.
I wanted a first term of 6.
Doubling is easy to do, so I did that. I realized that the sums of the first two columns would give 60 as the first term and 270 as the second.
I divided by 10 to get 6 as the first term.

Example 3 | Determining part of a whole

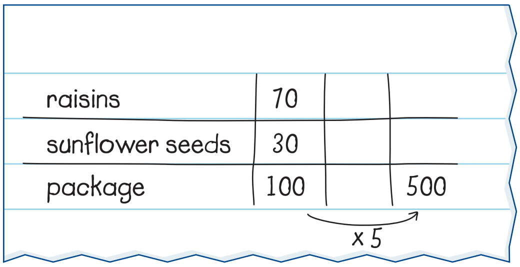
A bag of trail mix has 70 g of raisins for every 30 g of sunflower seeds. There are no other ingredients. In 500 g of trail mix, how many grams are raisins?

Misa's Solution



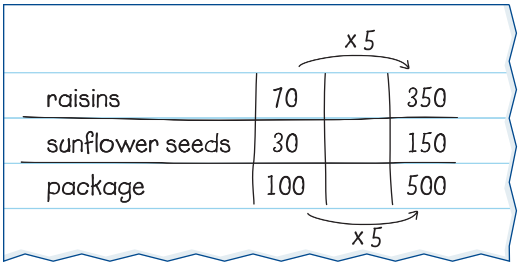
raisins	70		
sunflower seeds	30		
package			500

I used the first two rows of a ratio table to represent the parts; I added a third row to represent the whole. I did that since I knew about the parts for a smaller mixture, but not the whole amount for the package.



raisins	70		
sunflower seeds	30		
package	100		500

The whole for the small mixture was 100 g (70 g + 30 g).



raisins	70		350
sunflower seeds	30		150
package	100		500

I multiplied all the numbers by 5 since I wanted a total of 500.

There are 350 g of raisins in the package.

A Checking

1. Complete each ratio table.

a)

Boys	2	20	40		
Girls	3			15	45

b)

Bottles of juice	60				54
Bottles of water	90	9	27	99	

B Practising

2. Solve using a ratio table. Show your steps.

a) 2 : 3 = 36 : ■
b) 12 : 18 = 30 : ■
c) 6 : 8 = ■ : 44
d) 80 : ■ = 50 : 60

3. Mary made 2 L of orange juice from concentrate. She used 3 parts of water for each 1 part of concentrate. How much concentrate did she use?

4. Create two different ratio tables that would let you solve each proportion. Explain your thinking in creating the tables.

a) ■ : 20 = 63 : 84
b) 8 : 84 = 90 : ■
c) 8 : ■ = 20 : 35
d) 30 : 45 = ■ : 54

5. It takes 27 kg of milk to make 4 kg of butter.

a) How much milk is needed to make 3 kg of butter?
b) How much butter can you make from 540 kg of milk?

6. A map is drawn with a ratio of 3 : 2 000 000.

a) Why does the ratio 3 : 2 000 000 mean that 3 cm on the map represents 20 km? Use the fact that 1 km = 1000 m.
b) How many centimetres on the map represent 68 km?

7. A survey showed that residents of a city were 2 : 1 in favour of higher parking fines. In all, 4500 people were surveyed. How many were in favour of higher parking fines?

8. Two bags have the same ratio of red to blue marbles. The ratio is not 1:1. There are 9 red marbles in one bag and 16 blue marbles in the other.

a) How many of each colour might be in each bag?
b) Show that part a) has at least three other answers.

9. Barney says ratio tables are a good way to solve ratio problems since you can decide what to add, subtract, multiply, or divide to get the answer. What do you think Barney means?

Math **GAME**

Ratio Match

In this game, you make as many equivalent ratios as possible with six cards.

Number of players: 2–4

YOU WILL NEED

- a deck of playing cards with no face cards (aces represent 1s)

How to Play

1. Shuffle the cards and place them face down.
2. Each player takes six cards.
3. Make as many equivalent ratios as you can using your cards.
4. You get one point for each set of equivalent ratios that do not use the same four cards.
5. After six rounds, the player with the most points wins.

Nikita's Turn

I picked 4, 6, 7, 1, 1, and 2.

I created two pairs of equivalent ratios:

- 1 : 2 and 7 : 14
- 6 : 12 and 7 : 14

I got 2 points.

Preston's Turn

I drew 8, 10, 2, 1, 5, and 4.

I created four pairs of equivalent ratios:

- 1 : 4 and 2 : 8
- 4 : 5 and 8 : 10
- 5 : 10 and 1 : 2
- 2 : 5 and 4 : 10

I got 4 points.

3.3

Exploring Ratios with Three Terms

YOU WILL NEED

- Fraction Strips Tower, base ten blocks, counters, or play money
- chart paper
- coloured markers

GOAL

Use ratios to solve problems involving three values.

EXPLORE the Math

Allison, Nikita, and Misa pool their money to buy one lottery ticket. The **three-term ratio** 6 : 3 : 1 describes their shares of the ticket.

three-term ratio

a ratio that compares three quantities; e.g., the ratio 2 : 3 : 5 (or, 2 to 3 to 5) describes the ratio of red to blue to yellow squares.

🟥🟥🟦🟦🟦🟨🟨🟨🟨🟨

Brian, Preston, and Aaron also buy a ticket. The ratio 6 : 2.50 : 1.50 describes their shares of the ticket.

ENTER TO WIN
the Hospital Lottery
to buy new medical equipment

Tickets are only $10!

Grand prize	$10 000
2nd prize	$2500
3rd prize	$500

? If either ticket wins one of the prizes, how much should each person get?

Chapter 3

Mid-Chapter Review

Frequently Asked Questions

Q: When do you use ratios?

A: You use ratios when you want to compare quantities with the same units. For example

- the amount of water and concentrate needed to make orange juice
- the amount of time spent doing activities in a day
- the areas of two rectangles

Q: How can you use equivalent ratios to solve problems?

$$\frac{\text{clothes}}{\text{CDs}} = \underbrace{\frac{\$5}{\$2} = \frac{\square}{\$16}}_{\times 8}$$

$$\underbrace{\overbrace{\frac{\$5}{\$2} = \frac{\$40}{\$16}}^{\times 8}}_{\times 8}$$

A1: You can use equivalent fractions or set up a proportion to figure out an appropriate equivalent ratio. For example:

- You spent \$5 on CDs for every \$2 you spent on clothes. You spent \$16 on clothes. How much did you spend on CDs?

You multiply \$2 by 8 to get the amount spent on CDs, so multiply \$5 by 8 to get the amount spent on clothes.

You spent \$40 on clothes.

A2: You can use a ratio table and a given ratio to create an equivalent ratio. You can do this in several ways:

- Multiply or divide both terms in one column by the same amount.
- Add or subtract corresponding numbers in two or more columns to get one number in the equivalent ratio you need and read the table to get the other number.

8	4		36
18	9		81

÷2 ×9

For example, to solve $8 : 18 = \square : 81$, you might notice that 81 is not a multiple of 18, but it is a multiple of 9, which is also a factor of 18, so you try to get a 9 as the second term.

Practice

Lesson 3.1

1. **a)** List three ratios equivalent to 4 : 9.
   b) Can one term of a ratio equivalent to 4 : 9 be 360? Is there more than one way? Explain.

2. Calculate each missing term.
   a) ■ : 11 = 11 : 121
   b) 5 : 2 = ■ : 14
   c) ■ : 5 = 17 : 2.5

3. Green paint is mixed with white paint in the ratio 5 : 3 to make the green paint lighter.
   a) What fraction of the paint is white?
   b) There are 2 L of mixed paint. How many litres of green paint were used to make it?

4. Determine each ratio for the numbers from 1 to 100, including 1 and 100.
   a) the number of even numbers to the number of odd numbers
   b) the number of multiples of 6 to the number of multiples of 8

Lesson 3.2

5. Complete each ratio table.

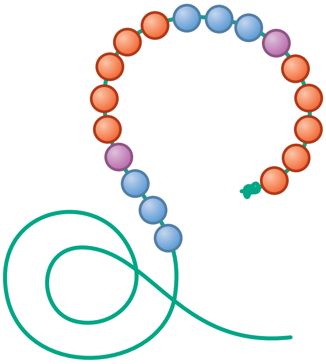
a)

Number of days	7			28	
Number of school days	5	35	100		40

b)

Number of dimes		9		6	
Value of dimes	10¢	90¢	80¢		800¢

6. The ratio of Grade 8 students to Grade 7 students on a field trip is 7 : 5. In all, 84 students went on the trip. Determine the number of Grade 8 students on the trip using a ratio table.



Lesson 3.3

7. Jerry makes bead necklaces. Each necklace has red, blue, and purple beads in the ratio 5 : 3 : 1. One necklace has 36 beads altogether. How many of each colour does it have?

3.4 Using Rates

YOU WILL NEED
- a calculator

GOAL

Use rates and equivalent rates to solve problems.

LEARN ABOUT the Math

Adam Sioui of Calgary won a gold medal in 2007 for swimming the 100 m backstroke in just under 56 s.

? On average, how long did Adam take to swim 1 m?

A. **Speed** is an example of a **rate.** It compares the distance travelled to the time in which the distance is travelled. How do you know that Adam's speed was less than two metres each second?

B. Describe how you would calculate Adam's speed as a **unit rate** of metres per second (m/s).

C. How do you know that Adam swam 1 m in less than 1 s?

D. What is Adam's unit rate in seconds per metre (s/m)?

Communication | *Tip*

The 1 in a unit rate is usually not written; e.g., 95.2 km/1 h is written as 95.2 km/h.

speed

the rate at which a moving object travels a certain distance in a certain time; for example, a sprinter who runs 100 m in 10 s has a speed of 100 m/10 s = 10 m/s.

rate

a comparison of two amounts measured in different units; for example, cost per item or distance compared to time. The word "per" means "to" or "for each" and is written using a slash (/); for example, a typing rate of 250 words/8 min.

unit rate

a rate in which the second term is 1; for example, in swimming, 12 laps/6 min can be rewritten as a unit rate of 2 laps/min

equivalent rate

a rate that describes the same comparison as another rate; e.g., 2 for $4 is equivalent to 4 for $8.

Reflecting

E. Why was your answer to part D an **equivalent rate** to 56 s/100 m?

F. How are rates like ratios? How are they different?

WORK WITH the Math

Example 1 | Using a proportion to solve a rate problem

At last year's school picnic, the helpers served about 160 L of lemonade to 250 people. About how much lemonade did each person have?

Aaron's Solution

The rate is 160 L/250 people.

$$\frac{\text{lemonade}}{\text{people}} = \frac{160\text{ L}}{250\text{ people}} = \frac{■\text{ L}}{1\text{ person}}$$

÷ 250

I want to know the amount of lemonade each person had: that is a unit rate. I used the information to write a proportion.

$160 \div 250 = 0.64$

Each person could have had 0.64 L, or 640 mL, of lemonade.

I realized that I had to divide 250 by 250 to get 1, so I had to divide 160 by 250. I used my calculator.

Example 2 | Calculate equivalent rates using a ratio table

Allison's favourite cereal comes in two sizes. The small box is 750 g and costs $3.99. The giant box is 2.5 kg and costs $12.49. How much does Allison save by buying the giant box?

Allison's Solution

2.5 kg = 2500 g

I converted 2.5 kg to grams since the mass of the other box was in grams.

Grams	750			2500
Cost	$3.99			

I used a ratio table to figure out the cost of the giant box at the small-box rate.

I noticed that, if I divided 750 by 3 and then multiplied by 10, I would get the number of grams for the big box.

$\div 3$ $\times 10$

Grams	750	250		2500
Cost	$3.99	$1.33		$13.30

So 2500 g would cost $13.30 at the small-box rate.

$13.30 – $12.49 = 81¢
I save 81¢ by buying the giant box.

I subtracted the cost for the giant box from what it would have cost at the small-box rate.

A Checking

1. Write two equivalent rates for each case. One of your rates should be a unit rate.
- **a)** 5 goals in 10 games
- **b)** 10 km jogged in 60 min
- **c)** 10 penalties in 25 games

2. On a hike, Peter walked 28 km in 7 h.
- **a)** What was his speed in kilometres per hour?
- **b)** How far would he walk in 2 h at that speed?

B Practising

3. Determine the missing term.

a) Three trucks have 54 wheels. Six trucks have ■ wheels.

b) In 5 h, you drive 400 km. In 2 h, you drive ■ km.

c) In 2 h, you earn \$20. In 9 h, you earn ■.

d) Six boxes contain 72 doughnuts. ■ boxes contain 48 doughnuts.

4. Brad pays \$56 for four CDs.

a) At this rate, how many CDs can he buy with \$42?

b) Why might you use a different strategy to figure out how many CDs he could buy with \$28?

5. Calculate the unit cost (the cost for 1 kg, 1 L, or 1 m^2).

a) \$3.70 for 2 kg of peaches

b) \$2.99 for 1.89 L of juice

c) \$283.28 for 22.5 m^2 of floor tiles

6. Wayne Gretzky scored 2857 points in 1487 NHL hockey games.

a) Calculate his average points per game to the nearest hundredth.

b) At this rate, how many more points would he have scored in 79 more games?

7. Suppose 6 kg of oranges cost \$14. How many kilograms of oranges can you buy for \$21?

8. A grey whale's heart beats 24 times in 3 min. How many times does it beat in a day?

9. When might you use the concept of rate in a grocery store?

10. Jason's mom drove 160 km to a stampede at 100 km/h. Then she drove back home at 90 km/h. What was the difference in time between the trips?

11. Create a problem that involves calculating speed, in which you know the distance travelled and the time taken to travel that distance. Solve the problem in two ways.

3.5 Communicate about Ratios and Rates

YOU WILL NEED
- a calculator

GOAL

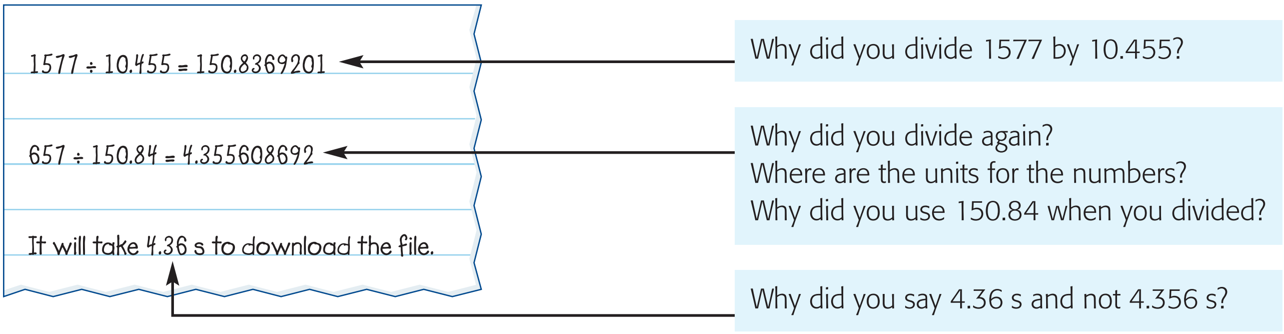
Explain your thinking when solving ratio and rate problems.

LEARN ABOUT *the Math*

Nikita asked Misa to check her explanation to this problem.

A computer downloads a 1577 KB file in 10.455 s. How long will it take to download a 657 KB file? Explain.

Nikita's Explanation



$1577 \div 10.455 = 150.8369201$

$657 \div 150.84 = 4.355608692$

It will take 4.36 s to download the file.

Misa's Questions

Why did you divide 1577 by 10.455?

Why did you divide again?
Where are the units for the numbers?
Why did you use 150.84 when you divided?

Why did you say 4.36 s and not 4.356 s?

❓ How can Nikita improve her explanation?

A. How can you answer Misa's questions to improve Nikita's explanation?

B. What other questions could Misa have asked?

Communication *Checklist*

- ✓ Did you explain how you performed your calculations?
- ✓ Did you explain why you did each calculation?
- ✓ Did you use a model, a chart, or a diagram to make your thinking clear?
- ✓ Did you check that your answer makes sense?

Reflecting

C. Which parts of the Communication Checklist did Nikita cover well? Which parts did Misa cover in her questions?

D. How else could you change Nikita's explanation to make it better?

WORK WITH the Math

Example 1 | Communicating about ratios

The Coyotes have won 8 of their first 20 soccer games. If this continues, how many games would you expect them to win out of 30 games? Explain your thinking.

Preston's Solution

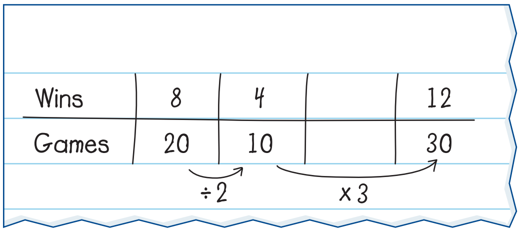
I realized this was a ratio problem and I assumed that the team's winning ratio would stay the same.

I set up a proportion in which the first value was wins and the second was total games.

$8:20 = \square:30$

I like ratio tables, so I used one to solve the problem.

It was hard to figure out what to multiply 20 by to get 30, so I divided 8 and 20 by 2 to get 4 and 10 and then I multiplied both terms by 3.



Wins	8	4		12
Games	20	10		30

In 30 games, they would probably win 12.

This answer is reasonable, since if they played 40 games, they would win 16, and 12 is between 8 and 16.

A Checking

1. You are asked to solve this problem:
Marlene can run 4 km in 30 min. Can she run 6 km in 45 min?
Complete the calculation and explanation. Use the Communication Checklist to help you.

$$\frac{4\text{ km}}{30\text{ min}} = \frac{\text{distance}}{\text{time}}$$

I know that Marlene runs 4 km in 30 min. I can write a proportion to show this information.

B Practising

2. Mahrie was given this problem to solve:
Sam has a part-time job delivering flyers. He earns 25¢ for every 10 flyers he delivers. He needs $45 to buy his mom's birthday gift. How many flyers must he deliver?

Mahrie wrote this. How would you improve her explanation?

45 x 4 = 180 → 1800 flyers

3. The capacity of a glass is 270 mL, and the capacity of a Thermos flask is 1 L. Akeem said the ratio 270 : 1 compares the capacity of the glass with the capacity of the Thermos. Is he correct? Explain.

golden raisins
$0.66/100 g

dark raisins
$0.55/100 g

4. Raisins in a bulk-food store are priced as shown.
- **a)** A recipe calls for 500 g of raisins. How much money will you save if you buy the cheaper raisins? Explain.
- **b)** Raj bought $3.63 worth of one kind of raisins. What was the mass in grams?
- **c)** Is there more than one answer to part b)? Explain.

5. Jake downloaded three files, all at the same rate. The 1600 KB file downloaded in 14 s. The other two files downloaded in 21 s and in 10.5 s. About how large are the other two files? Explain.

6. A school district report says its student-to-teacher ratio is 21:1. The district has 50 teachers and 1280 pupils. Is the report accurate? Explain.

7. Without solving, say which is greater, the solution to $8:10 = 20:\square$ or the solution to $8:10 = \square:20$. Explain.

8. In Ellen's school, four boys play sports for every three girls who play sports. Can there be exactly 80 girls who play sports?

CURIOUS Math

Birth Rates

This map shows the birth rate per 1000 people for Alberta, British Columbia, and the Northwest Territories in 2005. But does it tell you where the most babies were born?

1. Where do you predict the most and least babies were born? Why?
2. Why do you need to know the populations of those regions to be sure?
3. Using the table, calculate the approximate number of births in each region.

Region	Approximate population in 2005
Alberta	3.3 million
B.C.	4.3 million
N.W.T.	43 thousand

4. Does the highest birth rate mean the most births? Explain.

3.6

Using Equivalent Ratios to Solve Problems

YOU WILL NEED

- a calculator

GOAL

Solve rate and ratio problems using proportions and ratio tables.

LEARN ABOUT the Math

Allison and Nikita sorted their CD collections. The three-term ratio of rap CDs to pop CDs to rock CDs is 4 : 6 : 8. They have 63 pop CDs.

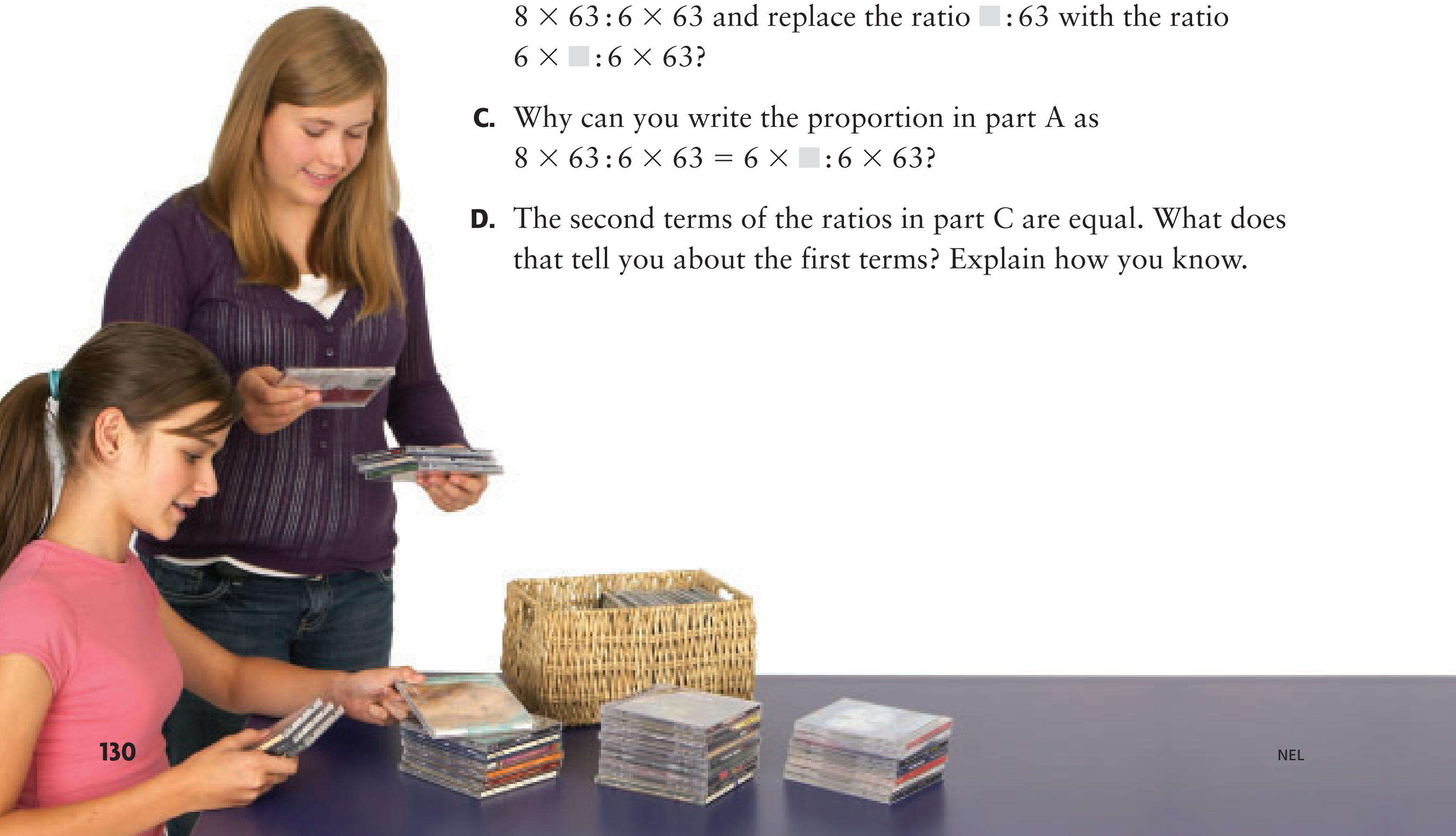
How many rock CDs and rap CDs do they have?

A. Why can you solve $8 : 6 = \blacksquare : 63$ to figure out the number of rock CDs?

B. Why can you replace the ratio 8 : 6 with the ratio $8 \times 63 : 6 \times 63$ and replace the ratio $\blacksquare : 63$ with the ratio $6 \times \blacksquare : 6 \times 63$?

C. Why can you write the proportion in part A as $8 \times 63 : 6 \times 63 = 6 \times \blacksquare : 6 \times 63$?

D. The second terms of the ratios in part C are equal. What does that tell you about the first terms? Explain how you know.



E. Why can you use the equation $8 \times 63 = 6 \times \blacksquare$ to solve for the number of rock CDs?

F. What proportion can you use to solve for the number of rap CDs?

G. What equation can you use to solve for the number of rap CDs?

H. Use the equations in parts E and G to calculate the number of rock CDs and rap CDs they have.

Reflecting

I. Why should the number of rap CDs be half the number of rock CDs?

J. Why might you call the method you used to solve the proportions "cross-multiplying"? Why does it work?

WORK WITH *the Math*

Example 1 | Solving part-to-part ratio problems

To make green paint, the paint store mixes yellow and blue paint in the ratio of 2 : 3. If they used 15 L of yellow paint, how much blue paint did they use?

Aaron's Solution

$\frac{2}{3} = \frac{15}{\blacksquare}$	To determine the amount of blue paint, I set up a proportion.
$\frac{2 \times \blacksquare}{3 \times \blacksquare} = \frac{15 \times 3}{\blacksquare \times 3}$	I can multiply each side of the equation by 1 without changing the equation. To get a common denominator, I multiplied the right side by $\frac{3}{3}$ and the left side by $\frac{\blacksquare}{\blacksquare}$. $\frac{1}{1} = \frac{3}{3} = \frac{\blacksquare}{\blacksquare}$
$2 \times \blacksquare = 15 \times 3$	I knew the numerators were equal since two fractions with the same denominator are only equal if their numerators are equal.
$\blacksquare = \frac{15 \times 3}{2}$ $= \frac{45}{2}$, or 22.5	I solved by dividing 3×15 by 2.

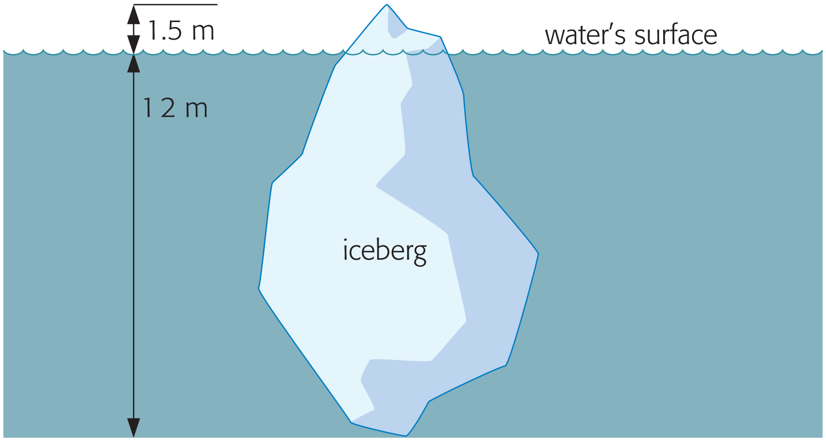
They used 22.5 L of blue paint.

A Checking

1. A girl who measures 50 cm in length at birth will probably grow to an adult height of 165 cm. Based on this, what will be the likely adult height of a girl whose length at birth is 48 cm?

B Practising

2. For every 1.5 m of an iceberg that is above the water, 12.0 m of the iceberg is below the water. Suppose 9 m of an iceberg is above the water. Is the iceberg 72 m in length from top to bottom? Explain.



3. A water tank holds 80 L. It is leaking at a rate of 1.5 L/min. How long will it take before these amounts leak out?

a) 5 L **b)** 20 L **c)** 35 L

4. About 22 out of 99 adult Canadians have difficulty reading. There are about 900 000 adults in Manitoba. How many adults in Manitoba might have difficulty reading?

5. A 13-year-old's heart might beat about 84 times per minute. About how long would it take her heart to beat 10 000 times?

6. The ratio of the running speed of cats to domestic pigs to chickens is 30:11:9. Approximately how many metres could a pig and a chicken run in the time a cat takes to run 1 km? 1 km = 1000 m.

7. On a 1035 km trip from Calgary, to Abbotsford, B.C., Dave's dad drove at 85 km/h. On the trip back, there were construction problems and his speed was 75 km/h. What was his average speed for the whole trip?

8. Digital televisions often have a width-to-height ratio of 16 : 9. The width of a TV screen is 104.0 cm. What is its height?

9. In 2005, 4 in 10 Canadian teens aged 12 to 17 were exposed to second-hand smoke. About how many students in your school would have been exposed to second-hand smoke in 2005?

10. In 2007, the amount of ozone over Antarctica dropped 30.5 million tonnes. In 2006, the amount of ozone dropped 44.1 million tonnes.

a) Describe the drop in the amount of ozone for the two years as a ratio.

b) If the drop in 2008 could be described by an equivalent ratio, what would the drop in tonnes be?

11. The population density of an area tells how many people there are for each square kilometre. For example, the population density of Canada is 3.2/km^2.

a) The population of Edson, Alberta, is 8098. What would its area be, based on the Canadian population density?

b) The actual area of Edson is 29.54 km^2. Is Edson more crowded or less crowded than Canada in general? Explain.

c) The population density of South Korea is 460/km^2. There are about 34 million people in Canada. How many people would live in Canada if it were as crowded as South Korea?

Reading Strategy

Finding Important Information

Use a KSO chart to help you solve the problem.
What do you **K**now?
What do you want to **S**olve?
What **O**ther information do you need?

12. On average, a certain baseball player has 212 hits in 1000 times at bat. Describe three different strategies you can use to determine how many hits he is likely to get in 400 times at bat.

13. Create and solve a problem that uses $\frac{2}{3}$ as a ratio and as a rate.

Chapter 3

Chapter Self-Test

1. **a)** Write two ratios equivalent to $5:9$, using only odd numbers.

   b) Write two ratios equivalent to $5:9$, each with one term of 90.

blue	3	24	27	54	51
yellow	4				
red	8				

2. Complete the ratio table in the margin.

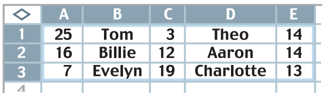
3. Calculate the missing term in each proportion.

   a) $2:11 = \blacksquare:55$

   b) $\frac{6}{\blacksquare} = \frac{10}{15}$

   c) $\frac{3}{4} = \frac{21}{\blacksquare}$

   d) $4.2:6.3 = \blacksquare:31.5$



◇	A	B	C	D	E
1	25	Tom	3	Theo	14
2	16	Billie	12	Aaron	14
3	7	Evelyn	19	Charlotte	13

4. A spreadsheet has cells that hold either numbers or words. There are three cells with numbers for every two cells with words. There are 250 cells in the spreadsheet. How many cells have numbers?

5. Nicole earned \$78 in 9 h. How much would she earn in 12 h?

6. The ratio of cats to dogs at an animal shelter is $5:2$. Currently, 63 cats and dogs are up for adoption. How many cats are up for adoption?

7. In Canadian universities, 6 out of every 10 graduates from a first degree are women. If about 160 000 students graduated last year, about how many were women?

8. Jason's mom drove 70 km to the city at 80 km/h. Then she drove back home at 90 km/h. What was the difference in time between the trips?

9. Three granola bars cost \$2.67. Use three strategies to figure out the cost of 10 bars.

What Do You Think Now?

Revisit What Do You Think? on page 105. Have your answers and explanations changed?

Chapter 3

Chapter Review

Frequently Asked Questions

Q: How can you solve a rate problem using an equivalent rate?

A: You can set up a proportion to figure out an appropriate equivalent rate. For example: You travel 650 km in 8 h. How far would you travel in 3 h?

$$\frac{650}{8} = \frac{\blacksquare}{3}$$

$$\frac{650 \times 3}{8 \times 3} = \frac{\blacksquare \times 8}{3 \times 8}$$

write the two rates as equivalents with a common denominator

$$650 \times 3 = \blacksquare \times 8$$

the numerators are equal

$$1950 = \blacksquare \times 8$$

$$1950 \div 8 = \blacksquare$$

$$243.75 = \blacksquare$$

You would travel 244 km in 3 h.

Q: How can you solve a ratio problem using a fraction?

A: You can set up an equation with two equivalent fractions. For example, to solve $5:6 = 12:\blacksquare$

$$\frac{5}{6} = \frac{12}{\blacksquare}$$

write the ratios as fractions

$$\frac{5 \times \blacksquare}{6 \times \blacksquare} = \frac{12 \times 6}{\blacksquare \times 6}$$

write the fractions as equivalents with the same denominator

$$5 \times \blacksquare = 6 \times 12$$

the numerators are equal

$$5 \times \blacksquare = 72$$

$$\blacksquare = 72 \div 5$$

$$= 14.4$$

Practice

Lesson 3.1

1. Write two equivalent ratios for each of the following.

a) 9 : 20 **b)** $\frac{4}{5}$ **c)** 21 to 3 **d)** 18 : 1.5

2. Solve each proportion.

a) 5 : 9 = 40 : ■ **b)** 30 : 80 = 51 : ■ **c)** ■ : 18 = 60 : 270

3. Draw a rectangle. Draw another rectangle twice as long and twice as wide. Describe each ratio.

a) ratio of the smaller diagonal to the larger one

b) ratio of the smaller area to the larger one

Lesson 3.2

Boys	15			5	
Girls	18	36	54		60

4. Complete the ratio table in the margin.

5. Solve the proportion 6 : 15 = 33 : ■ in more than one way using ratio tables. Show what you did.

Lesson 3.3

6. A stick is cut into four pieces of different lengths. The ratio of the length of one piece to the next shorter one is always 2 to 1. What is each piece as a fraction of the whole stick?

Lesson 3.4

7. Rewrite each rate as a number of items for $1.

a) 4 cookies for 50¢ **b)** 3 kg of sugar for $1.20

8. Which of the games on the left is the best deal? Explain.

Lesson 3.5

9. Sally asks, “If 20 + 5 = 25 and 30 + 5 = 35, then why does 20 : 30 not equal 25 : 35?” How would you answer Sally’s question?

Lesson 3.6

10. In a 750 g bag of salad mix, the ratio of lettuce to cabbage to carrots is 8 : 4 : 3 by mass. What is the mass of each ingredient?

11. A car odometer reads 14 985 km at the start of a trip, and the car has a full tank of gas. At the end of the trip, the odometer reads 15 235 km, and 30 L of gas is needed to refill the tank. How much gas does the car use per 100 km?

Chapter 3

Chapter Task

YOU WILL NEED

- a ruler
- a calculator

"If the world …"

Suppose you were writing a book in which each two-page spread began, "If the whole school (or town or city) were just like our classroom, then …"

Task | *Checklist*

- ✔ Did you use a variety of strategies to make your predictions?
- ✔ Did you explain how you performed your calculations?
- ✔ Did you explain why you did each calculation?
- ✔ Did you check that your predictions make sense?
- ✔ Did you make your results easy to understand?

? What would your pages say?

A. Select three items from the following list that apply to your classmates.

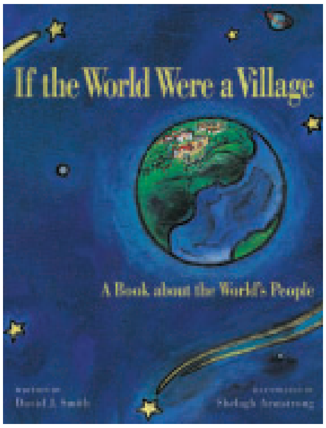
- the ratio of right-handed students to the total number of students
- the ratio of boys to the total number of students
- the ratio of students who climb stairs to get to their bedroom to those who do not
- the total number of minutes of TV that your classmates watched last night
- the approximate number of litres of water that your classmates drank yesterday

B. Find out how many students are in your school and how many people live in your community.

C. Use the information from parts A and B to create equivalent ratios for your three choices from part A for your whole school and your community.

D. Come up with two of your own ideas for ratios and rates that apply in your class and figure out how to use them to find out about your whole school or community.

E. Suppose your page said, "If the world were a village of 1000 people, there would be…." What would your page say for your five ratio choices?



Chapters 1–3

Cumulative Review

1. Which of the following is a perfect square?

A. 8
B. 9
C. 10
D. 11

2. Which digit is a possible last digit of a perfect square?

A. 2
B. 3
C. 5
D. 7

3. Which factor of 324 is its square root?

A. 4
B. 9
C. 18
D. 36

4. To one decimal place, the square root of a number shown on a calculator is 12.3. Which statement is true?

A. The number is between 150 and 155.
B. The number is between 155 and 160.
C. The number is less than 150.
D. The number is greater than 160.

5. Erik is flying a kite. Calvin is directly under the kite. The boys are 60 m apart, and the kite string is 100 m long. How high is the kite above Calvin?

A. 40 m
B. 80 m
C. 160 m
D. 6400 m

6. An airplane travels between two cities that are 1500 km apart. It climbs steadily from takeoff to cruise at 10 000 m during the first 200 km of the trip. It descends steadily from 10 000 m for 300 km to land. About what distance do climbing and descending add to the flight?

A. 0.4 km
B. 4 km
C. 10 km
D. 20 km

7. Ron's cookie recipe requires $2\frac{3}{4}$ cups of flour and makes 20 cookies. He wants to make 75 cookies. Which expression describes the amount of flour he should use?

A. $\frac{75}{20}$
B. $\frac{75}{20} + 2\frac{3}{4}$
C. $2\frac{3}{4} + \frac{75}{20}$
D. $2\frac{3}{4} \times \frac{75}{20}$

8. Which expression is equivalent to $3\frac{3}{8}$?

A. $\frac{17}{8}$
B. $\frac{24}{8}$
C. $\frac{27}{8}$
D. none of these

9. Which expression has the same value as $\frac{1}{2} \times \frac{2}{3}$?

A. $\frac{5}{6} \times \frac{2}{5}$
B. $\frac{3}{4} \times \frac{8}{9}$
C. $\frac{3}{8} \times \frac{4}{9}$
D. $\frac{3}{5} \times \frac{5}{6}$

10. Which expression has the greatest value?

A. $\frac{5}{6} \div \frac{2}{5}$ **B.** $\frac{5}{6} \times \frac{2}{5}$ **C.** $\frac{2}{5} \div \frac{5}{6}$ **D.** $\frac{6}{5} \times \frac{2}{5}$

11. Calculate $6\frac{1}{4} \div 2\frac{1}{2}$.

A. $2\frac{1}{4}$ **B.** $2\frac{1}{2}$ **C.** $3\frac{1}{8}$ **D.** $3\frac{1}{2}$

12. Calculate $\frac{3}{4} - \frac{5}{8} \times \frac{4}{5} + \frac{3}{5}$.

A. $\frac{7}{10}$ **B.** $\frac{1}{8}$ **C.** $\frac{5}{7}$ **D.** $\frac{17}{20}$

13. Which represents a ratio that is NOT equivalent to the others?

A. 4 to 5 **B.** 3 to 4 **C.** 8 : 10 **D.** 2 to 2.5

14. A candy mixture contains 55 g of chocolate-coated raisins and 45 g of chocolate-coated peanuts. What fraction of the mixture is peanuts?

A. $\frac{9}{11}$ **B.** $\frac{11}{9}$ **C.** $\frac{9}{20}$ **D.** $\frac{11}{20}$

15. Determine the missing terms in the proportion.

6 : ___ : 15 = ___ : 20 : 10

A. 1 and 25
B. 2.5 and 0.5
C. 10 and 12
D. 30 and 4

Vehicle Comparison

Vehicle	Gas used (L)	Distance travelled (km)
A	65	650
B	60	700
C	55	600
D	50	550

16. Which vehicle in the Vehicle Comparison chart has the best fuel efficiency?

17. Concrete is made by mixing cement, sand, and gravel in the ratio 1 : 3 : 4 by mass. Michael needs to make 160 kg of concrete. How many kg of gravel does he need?

A. 156 **B.** 80 **C.** 40 **D.** 20

18. Rena walked for $2\frac{3}{4}$ h and travelled $7\frac{1}{2}$ km. Which expression should you use to determine her speed?

A. $2\frac{3}{4} \times 7\frac{1}{2}$ **B.** $2\frac{3}{4} \div 7\frac{1}{2}$ **C.** $7\frac{1}{2} \times 2\frac{3}{4}$ **D.** $7\frac{1}{2} \div 2\frac{3}{4}$

19. Marie rides her bike at 10 km/h along the legs of the right-triangle path shown. Imelda walks along the hypotenuse at 2 km/h. How much of a head start should Imelda get so that the two girls arrive at the destination at the same time?

A. 72.5 min **B.** 24.0 min **C.** 21.6 min **D.** 12 min