







<p><b>TUESDAY</b></p>  <p>Cloudy periods High 17 °C POP 20%</p>	<p><b>TONIGHT</b></p>  <p>Cloudy periods Low 8 °C POP 20%</p>	<p><b>WEDNESDAY</b></p>  <p>Isolated showers High 21 °C Low 8 °C POP 30%</p>
<p><b>THURSDAY</b></p>  <p>Sunny with cloudy periods High 23 °C Low 11 °C POP 10%</p>	<p><b>FRIDAY</b></p>  <p>Mainly sunny High 20 °C Low 9 °C POP 10%</p>	<p><b>SATURDAY</b></p>  <p>Sunny with cloudy periods High 20 °C Low 7 °C POP 10%</p>



# Chapter 10

## *Probability*

### GOAL

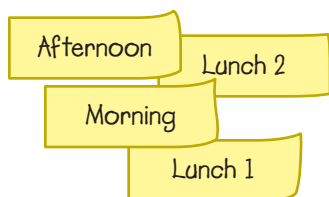
#### You will be able to

- determine the probability of two independent events
- verify a calculated probability using a different strategy
- solve problems involving the probability of independent events

◀ What do you think the probability is that it will rain on both Wednesday and Thursday?

**YOU WILL NEED**

- Spinners



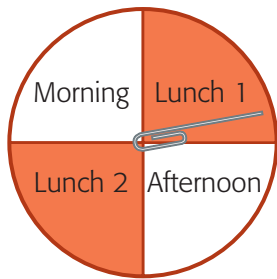
## Assigning Volunteer Times

Renée volunteered to be an office helper. There are four times when office helpers are needed.

Each month, the four volunteers pick a time period from a hat.

**?** What is the probability that Renée will get the afternoon time two months in a row?





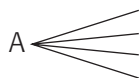
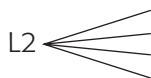
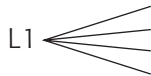
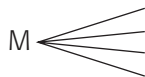
### tree diagram

a way to record and count all combinations of events, using lines to connect the two parts of the outcome

**A.** Renée made a spinner to model pulling slips of paper from a hat. She spun it twice and got “Afternoon” twice. Does this mean she will always be the afternoon office helper? Explain.

**B.** Complete this **tree diagram** to represent the possible outcomes of spinning for two different months.

1st month      2nd month

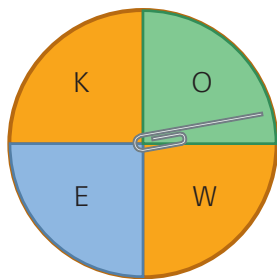


**C.** Determine the total number of possible outcomes for two spins.

**D.** Calculate the probability of Renée getting “Afternoon” two months in row. Use the tree diagram. Express the probability as a fraction and a percent.

## What Do You Think?

Decide whether you agree or disagree with each statement. Be ready to explain your decision.



### independent events

Two events are independent if the probability of one is not affected by the probability of the other.

- Adelle spins the spinner and gets a vowel. She is more likely to spin a consonant on her second spin.
- You cannot calculate the probability that two **independent events** will occur without using a tree diagram.
- If there are four possible outcomes for an event, then the probability of each outcome is a fraction with a denominator of 4.

# 10.1

## Exploring Independent Events

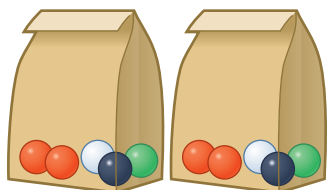
### YOU WILL NEED

- coloured marbles or counters
- two paper bags or boxes

### GOAL

**Investigate probabilities of independent events.**

### *EXPLORE the Math*



John and Taira place identical groups of coloured marbles into two bags. John chooses a marble from one bag and Taira chooses a marble from the other bag.

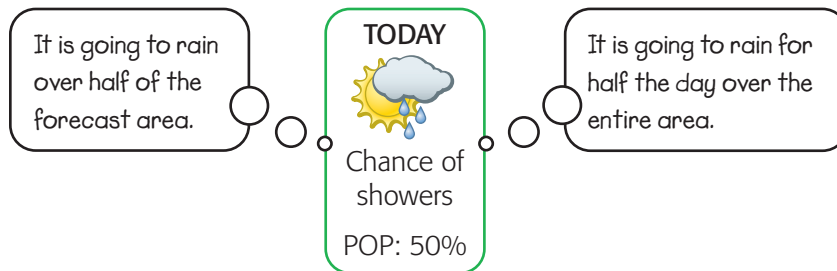
They do this for several different groups of marbles. Sometimes they use more marbles than other times.

**? How can you determine the probability that John and Taira will choose marbles that are the same colour?**



## Probability of Precipitation

There are many ideas about what meteorologists mean when they publish the probability that it will rain on a given day.



Officially, a probability of precipitation (POP) of 50% means that when similar weather conditions have been reported in the past, 50% of the time there has been rain within 12 hours of the forecast somewhere in the area.

For example, suppose a meteorologist collects data about the current weather around Edmonton. If she finds 50 similar situations, 10 of which resulted in rain somewhere in the area, then the POP is 20% for the next 12 hours.

1. Does rain falling in your forecast area mean that it will rain on your school?
2.
  - a) Make a tree diagram to show the outcomes of two consecutive days with a 50% POP for your area.
  - b) How likely is it to rain at least once?
  - c) How likely is it to rain both days?
  - d) How does the probability of rain both days in the forecast area compare to the probability of rain both days at your school?
3.
  - a) The POP is 100%. Will it rain today?
  - b) The POP is 0%. Will it rain today?
  - c) Suppose the POP published at 8:00 a.m. is 20% and you have a picnic at 11:00 a.m. Should you be surprised if it rains on your picnic? Explain.

# 10.2

## Probability of Independent Events

### YOU WILL NEED

- Spinners

### GOAL

Use pictures or charts to determine the probability of two independent events.

### *LEARN ABOUT the Math*

Holly and Ivan are playing a game. Both spin their spinners. Holly wins if the product is a multiple of 3 but not of 4. Ivan wins if the product is a multiple of 4 but not of 3. Otherwise, it is a tie.

**?** Who is more likely to win?



### outcome table

a chart that lists all possible outcomes of a probability experiment

### sample space

all possible outcomes in a probability experiment

### complementary event

the set of outcomes in the sample space in which the event does not happen; e.g., when rolling a die, the event (rolling 2) has the complementary event (rolling 1, 3, 4, 5, or 6).

### Communication **Tip**

You can write  $P(\text{Holly wins})$  as a shortcut for the words the probability of the event “Holly wins.”

- A.** Why are the results of spinning the two spinners independent events?
- B.** Complete the following **outcome table** for the **sample space** of this situation. The first few entries are filled in for you.

		Ivan's spin		
		3	4	5
Holly's spin	2	2, 3	2, 4	
	4			
	6			
	8			
	10			

- C.** Draw a tree diagram to verify the outcome table.
- D.** How many favourable outcomes are there for the event “Holly wins?” How many are there for the **complementary event** “Holly does not win?”
- E.** How many favourable outcomes are there for the event “Ivan wins?” How many are there for the complementary event “Ivan does not win?”
- F.** Which is greater,  $P(\text{Holly wins})$  or  $P(\text{Ivan wins})$ ? Explain how you know.

### Reflecting

- G.** How do you use the number of sections in each spinner to determine the number of rows and columns in the outcome table and the number of branches in the tree diagram?
- H.** How do you determine the total number of outcomes in the sample space using the outcome table or the tree diagram?
- I.** Why did it not matter whether you used an outcome table or a tree diagram to determine the probability of one of the students winning?



# WORK WITH the Math

## Example 1 Using an outcome table

John has all the red face cards from a standard deck and, Taira has all the black face cards from the deck. Each draws one card from their hand without looking.

- How many different outcomes are possible?
- Determine  $P(2 \text{ Jacks})$ .

### John's Solution

a)

		Taira's Cards					
		J♠	Q♠	K♠	J♣	Q♣	K♣
John's Cards	J♥						
	Q♥						
	K♥						
	J♦						
	Q♦						
	K♦						

There will be 36 outcomes in the table.

- b) The outcome table shows that there are 4 possible ways to choose a pair of Jacks.

		Taira's Cards					
		J♠	Q♠	K♠	J♣	Q♣	K♣
John's Cards	J♥	✓			✓		
	Q♥						
	K♥						
	J♦	✓			✓		
	Q♦						
	K♦						

$$P(2 \text{ Jacks}) = \frac{4}{36} = \frac{1}{9}$$

I made an outcome table.

I could see there would be 6 rows and 6 columns, so there had to be  $6 \times 6$  outcomes.

I used my outcome table and marked the 4 outcomes that had 2 Jacks.

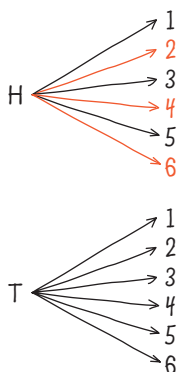
I decided not to use a tree diagram because there would be too many branches.

## Example 2 | Using a tree diagram

Suppose you toss a coin and then roll a six-sided die.

- What is the probability of getting heads and an even number?
- What is the probability of not getting heads and an even number?
- Verify your answers.

### Lam's Solution



I listed all the possibilities in a tree diagram. The 12 branches represent 12 equally likely outcomes.

- Three of the branches start with heads and end in an even number, so three outcomes are favourable.

$$P(H \text{ and even}) = \frac{3}{12}, \text{ or } \frac{1}{4}$$

- Not getting heads and an even number and the event in part a) are **complementary events**.

$$P(\text{not "H and even"}) + P(H \text{ and even}) = 1$$

$$P(\text{not "H and even"}) + \frac{1}{4} = 1$$

$$P(\text{not "H and even"}) = \frac{3}{4}$$

- I made an outcome table to verify my answers. There are three favourable outcomes for  $P(H \text{ and even})$  and there are nine favourable outcomes for  $P(\text{not "H and even"})$ .

$$P(H \text{ and even}) = \frac{3}{12}, \text{ or } \frac{1}{4}$$

$$P(\text{not "H and even"}) = \frac{9}{12}, \text{ or } \frac{3}{4}$$

	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

## Example 3 | Identifying events that are not independent

A bag contains 1 white marble, 1 blue marble, 1 green marble, and 1 red marble. Renée removes one marble and then another. Are these independent events?

### Renée's Solution

Since I did not put the first marble back in the bag, it was not possible for the first and second marbles to be the same colour. The result of the first draw has an effect on the second draw. The two draws are not independent events.

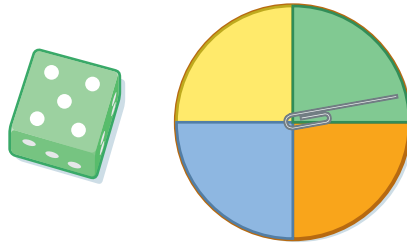


### A Checking

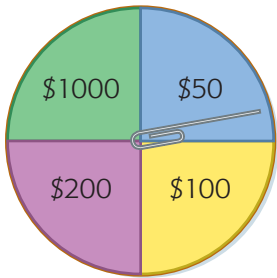
- Amit has two orange counters, two green counters, and two purple counters in a bag. She draws one counter from the bag and puts it back. She then draws another counter.
  - Make an outcome table that shows all possible outcomes in the sample space for this experiment.
  - Determine  $P(2 \text{ orange})$ .

### B Practising

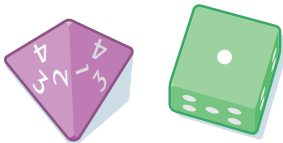
- Ellie uses the same counters that Amit used. She draws one counter from the bag and then another, without putting the first one back. Are these events independent or not? Explain.
- Suppose you roll the die and then spin the spinner.



- Use an outcome table, organized list, or tree diagram to show the sample space for this experiment.
- Determine  $P(3 \text{ and yellow})$ .
- Determine  $P(\text{odd number and green})$ .

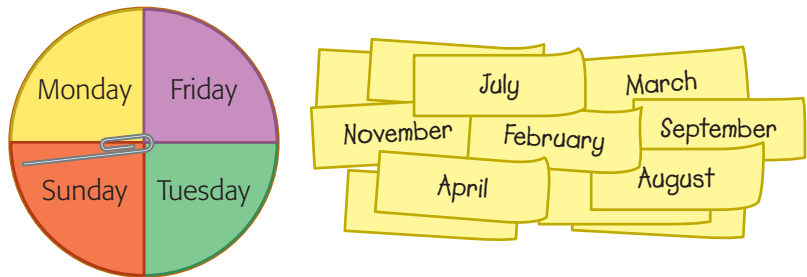


- Kaycee has won a contest. Her prize will be the sum of two spins of the prize spinner.
  - Use an outcome table to show Kaycee's possible winnings.
  - Determine  $P(\text{Kaycee wins more than } \$100)$ .
  - Determine  $P(\text{Kaycee wins less than } \$500)$ .

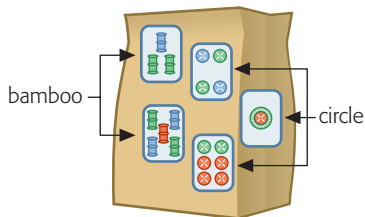


- Jenna rolls a four-sided die, numbered 1 to 4. Vi rolls a six-sided die. Determine the following probabilities.
  - $P(\text{both show } 2)$
  - $P(\text{both show an even number})$
  - $P(\text{both show a prime number})$
  - $P(\text{neither shows } 3)$
  - Use a different strategy to determine the sample space and verify your calculations for parts a) to d).

6. Rick spins the spinner and Donna draws the name of a month from a hat.



- Determine  $P(\text{Friday and April})$
- Determine  $P(\text{Monday and a month with 31 days})$
- Determine  $P(\text{Tuesday and a month starting with J})$
- Determine  $P(\text{Monday and not April})$
- Use a different strategy to determine the sample space and verify your calculations for parts a) to d).



7. Boris has a bag containing five mah-jong tiles: three from the circle suit  $\odot$  and two from the bamboo suit  $\text{||||}$ . He takes out a tile, puts it back, and then takes out another tile.
- Explain why selecting the first tile and selecting the second tile are independent events.
  - Determine  $P(\text{both circle tiles})$ .
  - Determine  $P(\text{both bamboo tiles})$ .
  - Determine  $P(\text{1st tile bamboo, 2nd tile circle})$ .
  - Use a different strategy to determine the sample space and verify your calculations for parts a) to d).
8. **a)** How can you use a tree diagram or an outcome table to determine the denominator of the probability of two independent events in fraction form?
- b)** Why is using a tree diagram or an outcome table a good strategy to calculate probabilities involving two independent events?

### Reading Strategy

#### Activating Prior Knowledge

Use what you already know about tree diagrams and outcome tables to help you solve this problem.

# 10.3

## Using a Formula to Calculate Probability

### GOAL

Develop and apply a rule to determine the probability of independent events.

### LEARN ABOUT the Math

Mrs. Wong gave her class a two-question quiz. There was one True/False question and one five-part multiple-choice question. The correct answers were False and D.

**QUIZ** Circle the correct answers.

1. The probability of an event may be represented by a fraction between 0 and 10.

TRUE

FALSE

2. Amit draws a card from a standard 52-card deck of playing cards. How many of these possible values for  $P(\text{drawing an Ace})$  are correct?

•  $\frac{1}{52}$

•  $\frac{4}{52}$

•  $\frac{1}{13}$

• about 0.077

• about 0.019

A. 0

B. 1

C. 2

D. 3

E. 4

**?** What is the probability that a student can correctly answer both questions by guessing?

## Example 1 | Using an outcome table

Create an outcome table to analyze the sample space.  
Then determine  $P(\text{both correct})$ .

### Renée's Solution

		Multiple-Choice Answers				
		A	B	C	D	E
True/False	True	T, A	T, B	T, C	T, D	T, E
Answers	False	F, A	F, B	F, C	<b>F, D</b>	F, E

$$\begin{aligned}
 P(\text{both correct}) &= \frac{1}{10} \\
 P(\text{both correct}) &= P(\text{correctly guessing T/F}) \times P(\text{correctly guessing M-C}) \\
 &= \frac{1}{2} \times \frac{1}{5} = \frac{1}{10}
 \end{aligned}$$

	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$
$\frac{1}{2}$					
$\frac{1}{2}$				<b><math>\frac{1}{10}</math></b>	

I made an outcome table in which the rows represented the answers to the True/False (T/F) question and the columns represented the answers to the multiple-choice (M-C) question.

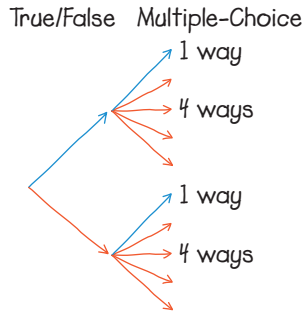
There were 10 outcomes and only one represented getting both questions correct.

I noticed that this is like calculating the fraction of the area of the chart that shows the favourable outcome. You calculate the area by multiplying the length by the width. The length of that section was  $\frac{1}{2}$  and the width was  $\frac{1}{5}$ .

## Example 2 | Using a tree diagram

Develop and apply a rule for calculating the probability of guessing the correct answer to the True/False question and the correct answer to the multiple-choice question.

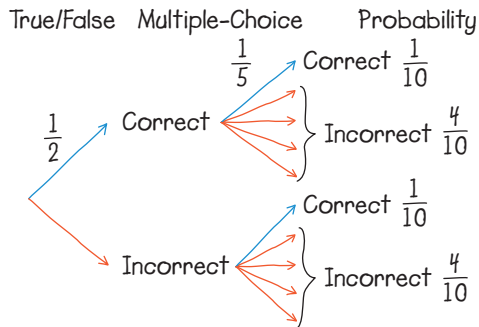
### Ivan's Solution



I drew a tree diagram that showed the number of ways to correctly (blue arrows) and incorrectly (red arrows) answer the questions.

I noticed that the number of ways of getting to an outcome on the right side of the tree was the product of the number of ways of getting to the end of the first branch and the number of ways of getting from there to the second branch.

$2 \times 5 = 10$  branches



I divided the number of ways of getting to an answer by the number of outcomes for that answer. That told the probability of each choice.

I saw that the probability of guessing both correct answers was the product of the probabilities of correctly guessing each one.

$$\begin{aligned}
 &P(\text{both correct}) \\
 &= P(\text{correctly guessing T/F}) \times P(\text{correctly guessing M-C}) \\
 &= \frac{1}{2} \times \frac{1}{5} = \frac{1}{10}
 \end{aligned}$$

### Reflecting

- How could you use Ivan's tree diagram to support Renée's reasoning?
- Why does it make sense that  $P(\text{getting both questions wrong}) = P(\text{getting first one wrong}) \times P(\text{getting second one wrong})$ ?

## WORK WITH the Math

### Example 3 | Calculating a combined probability

John draws a card from five cards, numbered 1 to 5. He puts the card back and then Renée draws a card. Calculate  $P(\text{both even})$ .

#### Solution A: Using an outcome table

		Renée's card				
		1	2	3	4	5
John's card	1					
	2		✓		✓	
	3					
	4		✓		✓	
	5					

Make an outcome table and place a checkmark in the outcomes where both cards were even numbers.

$$P(\text{both even}) = \frac{4}{25}$$

#### Solution B: Using a rule

$$\begin{aligned} P(\text{both even}) &= P(\text{1st card even}) \times P(\text{2nd card even}) \\ &= \frac{2}{5} \times \frac{2}{5} = \frac{4}{25} \end{aligned}$$

Since John put the card back, the probability of drawing an even card would not be affected by his first draw.

### Example 4 | Identifying events that are not independent

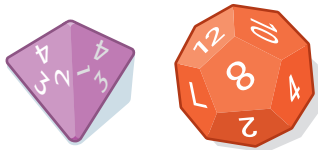
Angèle draws a card from five cards numbered 1 to 5 and does not put the card back. Then Taira draws a card. Show that you cannot multiply probabilities to determine  $P(\text{both even})$ .

#### Solution

$P(\text{both even})$  will have a denominator of 20.  $P(\text{even}) \times P(\text{even})$  has a denominator of 25. Since the denominators are different, the product of the probabilities is not the same as the probability of both cards being even.

It is not possible to have any outcomes where the same number is drawn twice, so there would be 20 outcomes in the table.  $P(\text{even})$  is a fraction with a denominator of 5.



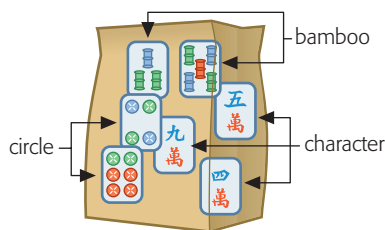


### A Checking

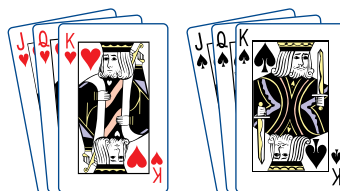
- Ron has a four-sided die, numbered 1 to 4, and a 12-sided die, numbered 1 to 12. He rolls both. Determine the following probabilities.
  - $P(\text{each shows a 4})$
  - $P(\text{each shows an even number})$
  - $P(\text{each shows a multiple of 4})$

### B Practising

- Bo has seven mah-jong tiles in a bag: two from the circle suit, two from the bamboo suit, and three from the character suit. He draws a tile, puts it back, and draws another tile.
  - Explain why the two draws are independent events.
  - Determine  $P(\text{both bamboo tiles})$ .
  - Determine  $P(\text{both character tiles})$ .
  - Determine  $P(\text{six-of-circles tile and the three-of-bamboo tile})$ .

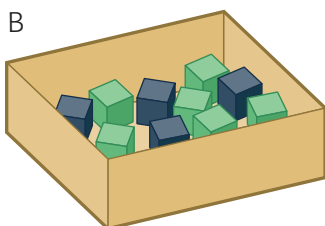
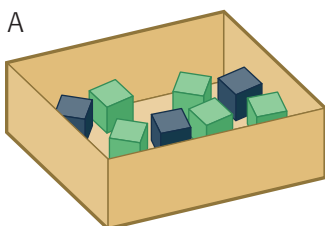


- Ram spins the spinner and then draws a card from those shown.

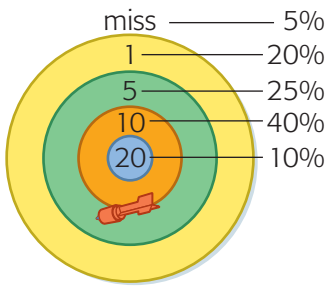


- Explain why the spin and the draw are independent events.
- Determine  $P(1, \text{Jack})$ .
- Determine  $P(\text{even number, King})$ .
- Determine  $P(\text{multiple of 5, black card})$ .

- One block is taken from box A. Then one block is taken from box B. Determine the following probabilities.
  - $P(\text{both blocks are green})$
  - $P(\text{both blocks are black})$
  - $P(\text{1st block is green and 2nd is black})$
  - $P(\text{1st block is black and 2nd is green})$



- Determine  $P(\text{both blocks are the same colour})$  for the two boxes in question 4.
  - Explain why you cannot multiply probabilities to answer part a).



6. Annie is a good dart thrower. The dart board shows the probability that she will hit each ring of the target. She throws two darts.
- Determine  $P(\text{both darts miss the target})$ .
  - Determine  $P(10 \text{ points with each dart})$ .
  - Determine  $P(10 \text{ points with first dart and } 20 \text{ points with the second})$ .
  - Discuss whether or not the results of the two throws are independent events.
7. Rishi has a combination lock for his locker. The lock uses two numbers between 0 and 39. The same number may be used twice. His combination is 35 – 24. Show how to determine the probability that someone could correctly guess the combination.
8. A school soccer team has the following history.
- They win 50% of the games played on rainy days.
  - They win 60% of the games played on fair days.
- The weather forecast says there is an 80% chance of rain for the next game day.
- Determine  $P(\text{it rains and they win})$ .
  - Determine  $P(\text{it does not rain and they win})$ .
  - Explain why the probabilities in parts a) and b) do not add to 100%.
9. **Situation 1:** David has 1 red and 3 green jellybeans in a bag, and Darlene has 1 red and 2 green jellybeans in a bag. David and Darlene each take one jellybean.
- Situation 2:** Bill has 2 red and 5 green jellybeans in his pocket. Bill takes 2 jellybeans from his pocket.
- Why can you use the multiplication rule for probabilities to determine  $P(\text{David and Darlene choose red})$ ?
  - Why can you not use the multiplication rule to determine  $P(\text{Bill chooses 2 red jellybeans})$ ?

# 10.4

## Communicate about Probability

### YOU WILL NEED

- Spinners
- assorted dice
- playing cards
- coloured cubes or marbles

### GOAL

**Communicate strategies for determining and verifying probabilities.**

### *LEARN ABOUT the Math*

Lam's baseball coach told him that his season batting average was 0.250. Lam was worried because he was out two at-bats in a row.

Taira wrote him a note explaining why he should not be too worried. She asked Holly to look it over before she gave it to Lam.

**? How can Taira make her note more convincing?**



## Taira's Note

## Holly's Comments

Lam, your batting average is 0.250. That means the probability that you will be put out in an at-bat is 0.750.

How do you know this is true?

The probability you will be put out two times in a row is:

$P(\text{out in the 1st at-bat}) \times P(\text{out in the 2nd at-bat})$

$$= 0.750 \times 0.750 = 0.5625$$

Are the two at-bats independent events?

This is more than 50%. So I think you should not be surprised that you were out in those two at-bats.

I found a spinner simulation on the Internet and used it to model 100 pairs of at-bats. I made two 4-section spinners and spun them 100 times.

Is this simulation a good model for Lam's at-bats?

Orange meant a hit, and any other colour meant an out.

### Spinner Simulation

Pick the colours for Spinner #1

Pick the colours for Spinner #2

Number of spins:  →

---

Spinner #1: ▲▲▲▲      Spinner #2: ▲▲▲▲

Spins: 100

1.▲▲	2.▲▲	3.▲▲	4.▲▲	5.▲▲	6.▲▲	7.▲▲	8.▲▲	9.▲▲	10.▲▲
11.▲▲	12.▲▲	13.▲▲	14.▲▲	15.▲▲	16.▲▲	17.▲▲	18.▲▲	19.▲▲	20.▲▲
21.▲▲	22.▲▲	23.▲▲	24.▲▲	25.▲▲	26.▲▲	27.▲▲	28.▲▲	29.▲▲	30.▲▲
31.▲▲	32.▲▲	33.▲▲	34.▲▲	35.▲▲	36.▲▲	37.▲▲	38.▲▲	39.▲▲	40.▲▲
41.▲▲	42.▲▲	43.▲▲	44.▲▲	45.▲▲	46.▲▲	47.▲▲	48.▲▲	49.▲▲	50.▲▲
51.▲▲	52.▲▲	53.▲▲	54.▲▲	55.▲▲	56.▲▲	57.▲▲	58.▲▲	59.▲▲	60.▲▲
61.▲▲	62.▲▲	63.▲▲	64.▲▲	65.▲▲	66.▲▲	67.▲▲	68.▲▲	69.▲▲	70.▲▲
71.▲▲	72.▲▲	73.▲▲	74.▲▲	75.▲▲	76.▲▲	77.▲▲	78.▲▲	79.▲▲	80.▲▲
81.▲▲	82.▲▲	83.▲▲	84.▲▲	85.▲▲	86.▲▲	87.▲▲	88.▲▲	89.▲▲	90.▲▲
91.▲▲	92.▲▲	93.▲▲	94.▲▲	95.▲▲	96.▲▲	97.▲▲	98.▲▲	99.▲▲	100.▲▲

In 100 spins, I got 56 pairs with no orange. That is close to what I calculated the probability of two outs in a row to be, so I think I am right.

Is one set of 100 spins enough to verify your calculations?

Do not worry about the two outs. That is not unusual.

Why is your conclusion reasonable?

## Communication Checklist

- ✓ Did you clearly state all your assumptions?
- ✓ Did you justify your assumptions and calculations?
- ✓ Did you justify your conclusions?

## Reflecting

- Which of Holly's comments do you think is most important for making Taira's note more convincing?
- What additional suggestions would you provide to help Taira make her note more persuasive?

## WORK WITH the Math

### Example 1 | Justifying a probability calculation

Angèle and Holly are planning a school picnic. If it is raining on the Thursday they have chosen, the event will move to the next day. The weather forecast says there is an 80% chance of rain on the scheduled day and a 40% chance of rain the next day.

Holly has written a letter to the principal about whether they should reschedule the event to another week. She has asked Angèle to read and edit it before she sends it.

### Angèle's Editing

The probability it will rain both days is:	←	Are these independent events?
$P(\text{rain on the 1st day}) \times P(\text{rain on the 2nd day})$		
$= 0.80 \times 0.40 = 0.32$	←	A tree diagram or outcome table might help.
However, it seems likely it will rain on Thursday. A 40% chance of rain on Friday is pretty high. If it does rain, we will have to store all the food and refreshments over the weekend.		
I think the risk is too great and we should wait until we can be guaranteed better weather.	←	Good point. It is not just about the probability being less than $\frac{1}{2}$ . Good plan, and you gave good reasons.

## A Checking

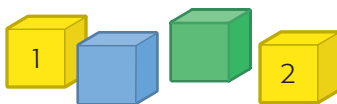
1. Lam's batting average is 0.250. Lam says that means, if he gets a hit in his first at-bat, he is sure to be out in his second at-bat. What do you think about Lam's interpretation of his batting average?

## B Practising

2. Calvin wrote a letter about whether the band should reschedule their spring picnic to another weekend. What questions would you ask Calvin to improve his letter?

The probability of rain on Saturday is 20%, and the probability of rain on Sunday is 60%. Since the chances of rain on each day are independent events, the probability it will rain both days is:
$P(\text{rain on Saturday}) \times P(\text{rain on Sunday}) = \frac{2}{10} \times \frac{6}{10}$
$= \frac{12}{100}, \text{ or } 12\%$
I think it is safe to have the picnic this weekend.

3. Two yellow cubes, one blue cube, and one green cube are placed in a bag. They are mixed up, and one cube is removed. It is returned to the bag, and the cubes are mixed again. A second cube is removed.



- a) Explain how to calculate  $P(\text{drawing 2 yellow cubes})$ .
- b) Show how to verify the probability you calculated in part a) using a different strategy.



4. Suppose you toss a coin and then roll a 12-sided die numbered 1 to 12.
  - a) Explain how to determine  $P(\text{heads, even number})$ .
  - b) Explain how to verify the probability you calculated in part a) using a different strategy.
5. A dresser drawer contains two socks of each colour: grey, black, and white. Without looking, you reach in, take one sock, and then take another. Explain why  $P(\text{both socks are grey})$  is not  $\frac{2}{6} \times \frac{2}{6}$ .
6. Suppose you have a 30% chance of catching a trout for every day you fish on Rainbow Lake.



- a) Calculate  $P(2 \text{ trout in } 2 \text{ days})$ .
  - b) Show how to use a different strategy to verify your calculation in part a).
  - c) Do you think that the result of your first day fishing and the result of your second day fishing are independent events? Explain.
7. A and B are two independent events. If  $P(A) = \frac{3}{4}$  and  $P(A \text{ and } B) = \frac{9}{20}$ , explain how to determine  $P(B)$ .
8. Suppose you calculate the probability that one event and then another will occur.
  - a) Why is showing that events are or are not independent an important part of your explanation of the calculation?
  - b) Why is doing the calculation a different way sometimes a good idea?

**YOU WILL NEED**

- two dice

## The Game of Pig

The goal of this game is to be the first player to score 100 points without rolling double sixes.

Number of players: 2 or more

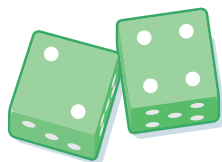


### How to Play

1. Players take turns rolling two dice as often as they wish or until they roll a 6.
2. If a player rolls a single 6, the turn ends and the score for the turn is 0. If a player rolls a double 6, the turn ends and all the player's points are lost. If a player stops before rolling a 6, the points for the score for the turn are the total of all rolls on that turn. Add this to the total from previous turns.
3. The first player to reach 100 points wins.

### Renée's Turn

I rolled a total of 6 and decided to roll again.



I rolled a total of 4 and decided to stop.

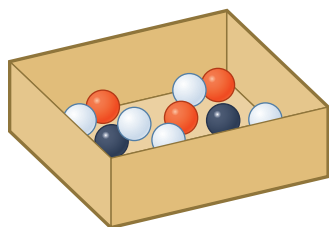
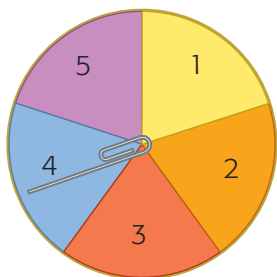


My score for the turn is  $6 + 4 = 10$ .

I add that to *my* total. I am now in second place!

Lam	Renée	Angèle
9	0	5
9	8	0
9	8	14
23	18	

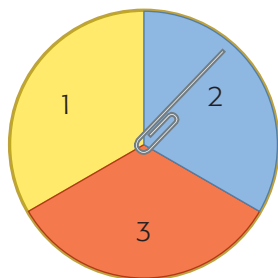




1. You spin the spinner twice.
  - a) Make an outcome table to describe the sample space.
  - b) Determine  $P(\text{even, even})$  and  $P(\text{odd, odd})$ .
  - c) Why are the probabilities different?
  
2. A multiple-choice quiz has five choices for each question. The correct answers to the first two questions are A and C.
  - a) Why is it reasonable to assume that guessing an answer to the second question is independent of guessing an answer to the first?
  - b) Calculate  $P(\text{guessing both correct answers})$ .
  - c) Use an outcome table or tree diagram to verify the probability.
  
3. A lacrosse team wins 70% of its games played on sunny days and 40% of its games played on rainy days. The weather forecast indicates an 80% chance of rain today. Determine the probabilities of these combinations of events.
  - a) It will rain and the team will win.
  - b) It will be sunny and the team will lose.
  - c) Describe a strategy you could use to verify your calculations.
  
4. Without looking, Yunjin removes a marble from the box, returns it, and then removes another. Determine each probability.
  - a)  $P(\text{both marbles are white})$
  - b)  $P(\text{1st marble is white, and 2nd is not red})$
  - c)  $P(\text{neither marble is black})$

### What Do You Think Now?

Revisit What Do You Think? on page 419. How have your answers and explanations changed?



## Frequently Asked Questions

**Q:** How can you calculate the probability of two independent events?

**A1:** You could examine a tree diagram, organized list, or outcome table for the events. The numerator of the probability will be the number of favourable outcomes, and the denominator will be the number of equally likely outcomes in the sample space.

For example, suppose you spin this spinner twice and wanted to know the probability of both spins having the same number. You could make an outcome table to show the sample space.

		Second spin		
		1	2	3
First spin	1	1, 1	1, 2	1, 3
	2	2, 1	2, 2	2, 3
	3	3, 1	3, 2	3, 3

The favourable outcomes are (1, 1), (2, 2), and (3, 3).

$P(\text{both spins are the same number})$

$$= \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

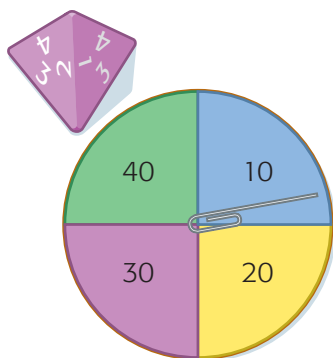
**A2:** You can multiply the probabilities of independent events to determine the probability that they will both occur.

For example, suppose you spin the above spinner twice.

$P(\text{1st spin is 2, and 2nd spin is odd})$

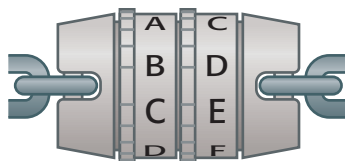
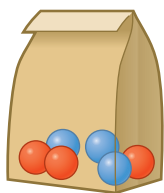
$$\begin{aligned} &= P(\text{1st spin is 2}) \times P(\text{2nd spin is odd}) \\ &= \frac{1}{3} \times \frac{2}{3} = \frac{2}{9} \end{aligned}$$

## Practice



### Lesson 10.2

- You toss a four-sided die, numbered 1 to 4, and spin the spinner.
  - Use an outcome table to show all possible combinations of die tosses and spins.
  - Determine  $P(1, 20)$ .
  - Determine  $P(\text{toss} > 1, \text{spin} < 30)$ .
  - Draw a tree diagram and use it to verify your answers.
- A paper bag contains three blue marbles and three red marbles. You remove a marble from the bag and then put it back. Then you remove another marble from the bag.
  - List all the outcomes.
  - Calculate  $P(\text{marbles are different colours})$ .
  - Would the events still be independent if the first marble were not returned to the bag before the second is removed? Explain.



### Lesson 10.3

- A combination lock has two wheels with the letters A to F. The lock combination comes preset from the factory.
  - Calculate the probability of guessing the correct combination on the first try.
  - Use a different strategy to verify the probability calculation.
- A provincial park claims that the probability of spotting moose in the park on any day is 0.15 and the probability of spotting loons is 0.70.
  - Determine  $P(\text{moose and loon on the same day})$ .
  - Determine  $P(\text{moose on two consecutive days})$ .
  - Determine  $P(\text{no moose for two days})$ .
  - What assumptions did you make?

### Lesson 10.4

- Melik dropped a coin out of each of his pockets. He had one nickel, one dime, and six quarters in the left pocket and eight nickels and four quarters in the right pocket. Explain how to determine the probability that Melik dropped the greatest amount of money possible.

**Task** | Checklist

- ✓ Did you explain why you thought your prediction was reasonable?
- ✓ Did you justify the model you chose for your experiment?
- ✓ Was your experiment enough to make you confident about verifying the calculation?
- ✓ Did you justify your reasoning for Holly's probabilities?

**Free Throw**

Angèle can sink 60% of her shots from 5 m and 40% from 10 m. She will be required to shoot from each distance as part of the tryout for the school basketball team.

**?** How can you predict the probability that she will sink both shots?

- A. Show how to calculate the probability that she will make both shots. Justify the use of your strategy.
- B. Conduct an experiment to model Angèle shooting one shot from each of the two distances. Repeat the experiment several times. Do your experimental results verify your calculation in part A? Explain.
- C. How else can you verify your calculation in part A? Explain.
- D. Holly has the same combined probability of making her two shots as Angèle, but her probabilities for each distance are not the same as Angèle's. Determine two possible combinations that could describe Holly's probability of making a shot from each distance. Justify your strategy.

