Chapter 5

Measurement

GOAL

You will be able to

- create and use nets to construct prisms and cylinders
- develop strategies to calculate the surface area of prisms and cylinders
- develop formulas to calculate the volume of prisms and cylinders
- solve problems that involve the surface area and volume of prisms and cylinders

◀ What different shapes of boxes could you use to pack each item?

Chapter 5

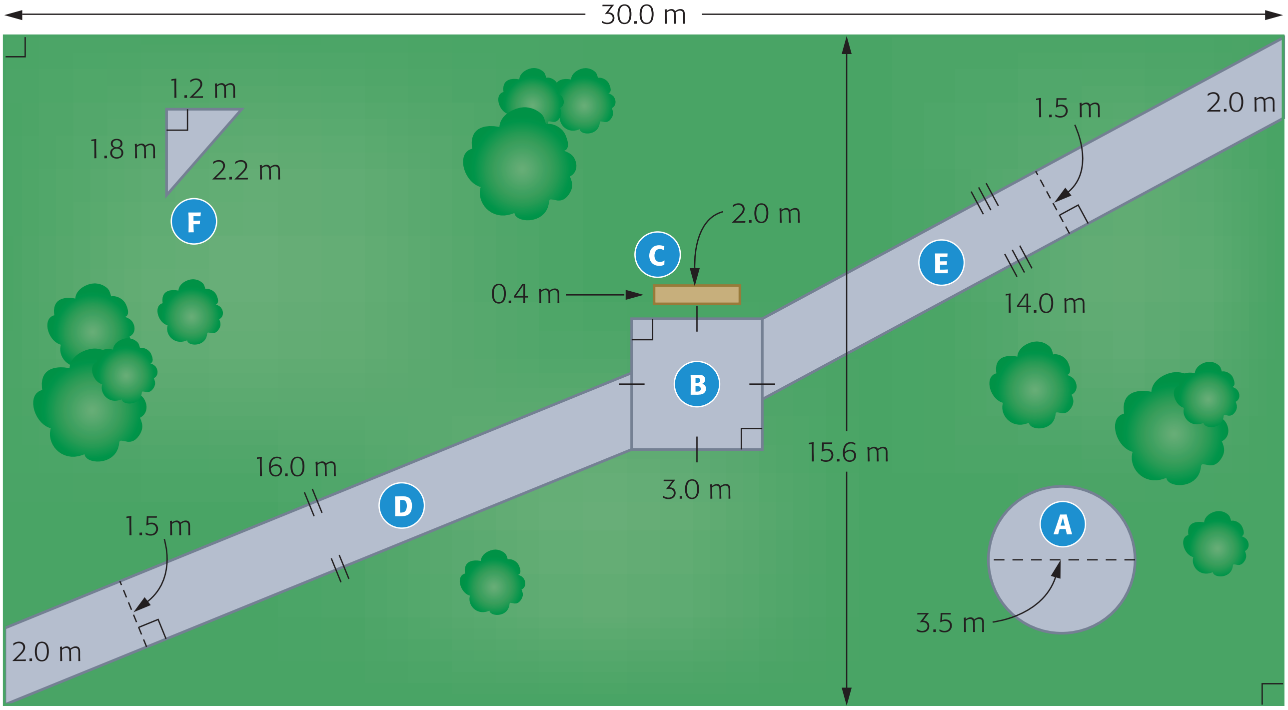
Getting Started

YOU WILL NEED

- 1 cm Grid Paper
- a ruler
- a calculator

Planning a Park

Allison has designed this park for her neighbourhood. The residents have asked that 80% of the park be grass.



A a patio	**C** a bench	**E** a path
B a central square	**D** a path	**F** a base for a drinking fountain

❓ Will Allison's design have enough grassy area?

A. What is the total area of the park?

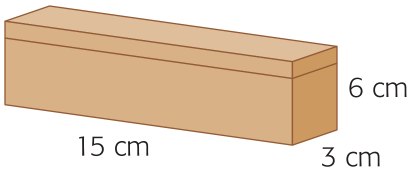
B. What area does each feature occupy?

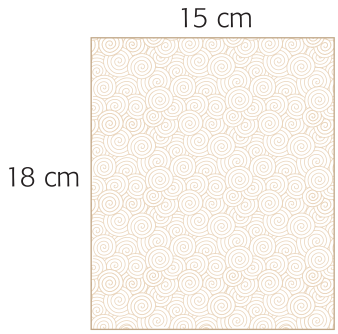
C. What percent of the park will be grass?

What Do You Think?

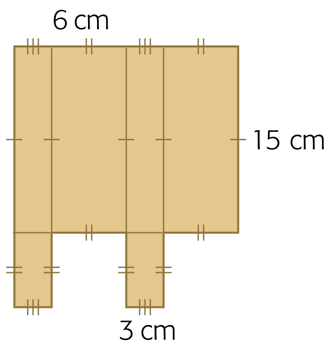
Decide whether you agree or disagree with each statement. Be ready to explain your decision.

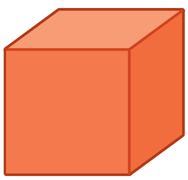
1. There is enough paper to cover all six faces of this box.





2. You can build the box in question 1 from this design.





3. If you double the length of each side of a cube, you double the total area of its faces.

4. If you double the length of each side of a cube, you double its volume.

5.1 Exploring Nets

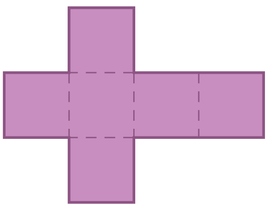
YOU WILL NEED
- Nets of Buildings I–VI
- 1 cm Grid Paper
- a ruler
- scissors
- tape

GOAL

Build 3-D objects from nets.

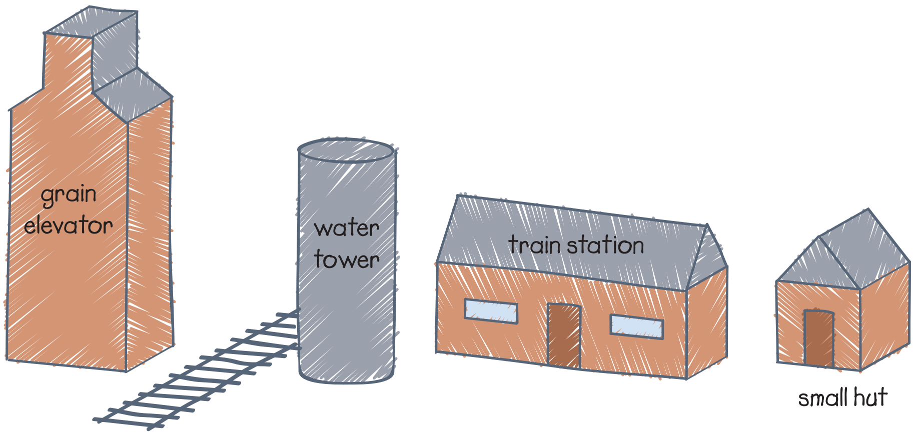
net

a 2-D pattern you can fold to create a 3-D object; for example, this is a net for a cube:



EXPLORE the Math

Brian wants to add a train station, a grain elevator, a water tower, and a small hut to his model railroad. He has the **nets** of five buildings, but they are not labelled.



? Which nets can Brian use to construct the buildings?

5.2 Drawing the Nets of Prisms and Cylinders

YOU WILL NEED

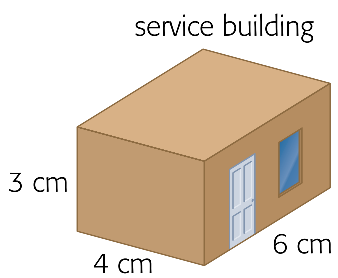
- 1 cm Grid Paper
- scissors
- transparent tape
- a compass

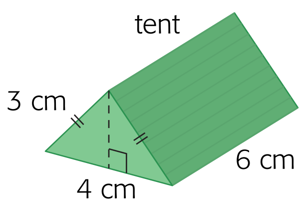
GOAL

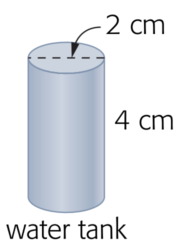
Draw nets of prisms and cylinders.

LEARN ABOUT the Math

Nikita is building a model campground. She plans to make the service building using a rectangular prism, tents using triangular prisms, and the water tank from a cylinder. She asked Misa to help her make nets for the models.







? How can Nikita and Misa draw nets of the models?

A. Draw the floor of the service building.

B. Draw the four walls around the floor and then the roof of the building. Draw them so the net folds to make a rectangular prism. Label the net with its dimensions.

C. Draw the floor of the tent. Draw the other four faces around it to make a net of the tent.

D. Draw the top of the water tank. Below it, draw the side of the water tank as if it were laid out flat. Draw the bottom of the water tank below that.

E. Cut out, fold, and tape your nets to make the models.

Reflecting

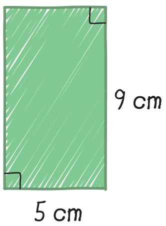
F. When you drew each net, how did you decide where to place each face in relation to the others?

WORK WITH the Math

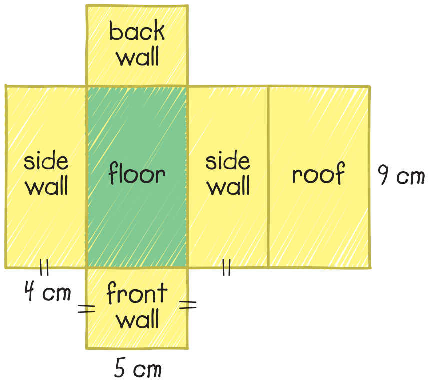
Example 1 | Creating a net for a rectangular prism

Nikita wants to create a net of a model of a general store in her model campground. It is 9 cm long, 5 cm wide, and 4 cm high.

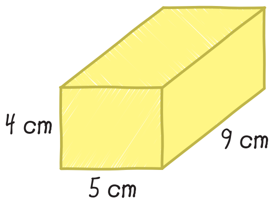
Nikita's Solution



I started with the floor. I drew a rectangle 5 cm wide by 9 cm long.



I drew the walls so they touched the floor and I drew the roof to touch one of the walls.

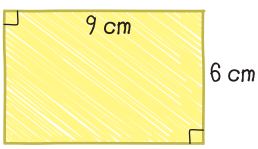


I know my net was correct, because when I folded it, the store was 9 cm long, 5 cm wide, and 4 cm high.

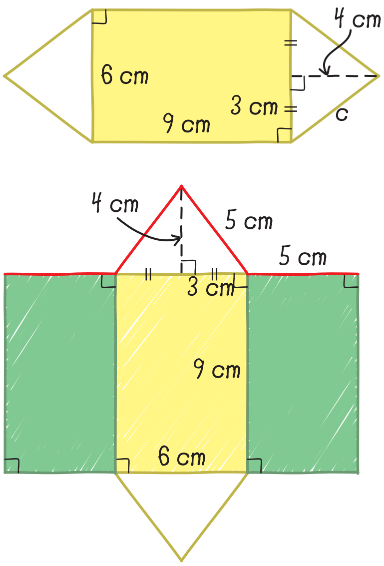
Example 2 | Creating a net for a triangular prism

Preston wants to create a net of a model of a large tent for the campground. It is 9 cm long, 6 cm wide, and 4 cm high.

Preston's Solution



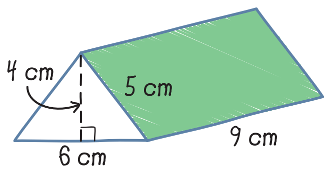
I started with the floor. I drew a rectangle 6 cm wide by 9 cm long.



$$c^2 = a^2 + b^2$$
$$= (3\text{ cm})^2 + (4\text{ cm})^2$$
$$= 25\text{ cm}^2$$
$$c = 5\text{ cm}$$

I needed two triangular walls. I knew each wall had a base of 6 cm and a height of 4 cm. I split one of the triangular walls into two right triangles. To figure out the side lengths of the walls, I used the Pythagorean theorem with one of the right triangles.

I drew the other walls. Each one was 5 cm wide by 9 cm long.

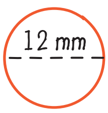


I know my net was correct, because when I folded it, the tent was 9 cm long, 6 cm wide, and 4 cm high.

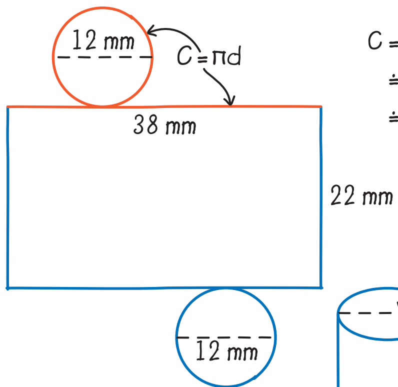
Example 3 | Creating a net for a cylinder

Allison is building a model fuel storage tank in the shape of a cylinder for the campground. It must be 12 mm in diameter and 22 mm high.

Allison's Solution

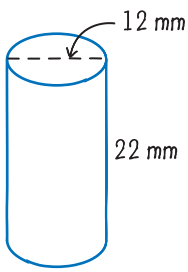


I drew the top of the cylinder. I could have started with the side, too.



$$C = \pi d$$
$$\doteq 3.14 \times 12$$
$$\doteq 38\text{ mm}$$

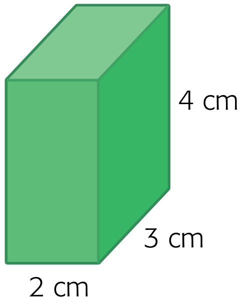
I drew the curved side laid out flat, as a rectangle. I knew it was 22 mm high. To determine its width, I calculated the circumference of the circle. I drew another circle for the base on the bottom.



I know my net was correct, because when I folded it, the storage tank was 22 mm high and 12 mm in diameter.

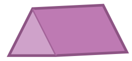
Communication | *Tip*

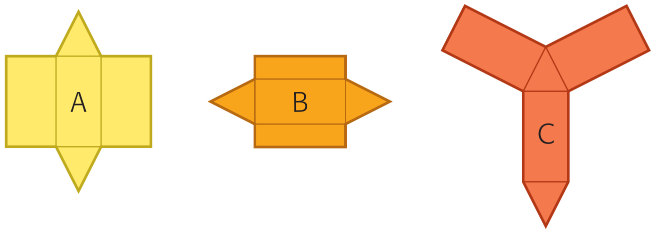
The symbol "≐" means approximately equal to.

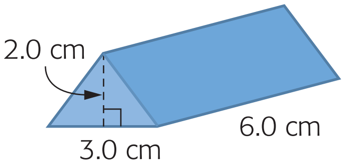


A Checking

1. Draw a net of the prism on the left.

2. Which net(s) will fold to make this prism? 

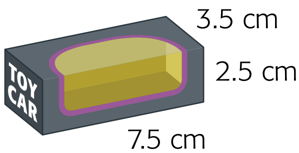




B Practising

3. Draw a net of the prism on the left.

4. Draw a net for each container.

   a) 

   b) 

   c) 

5. **a)** I have a rectangle in the middle, with a triangle attached to each of the two short sides and a rectangle attached to each of the two long sides. What net am I?

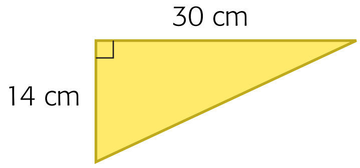
   b) I am made up of six congruent squares attached by their sides to form a T. What net am I?

6. **a)** What two prisms could you use to make this model of a house?

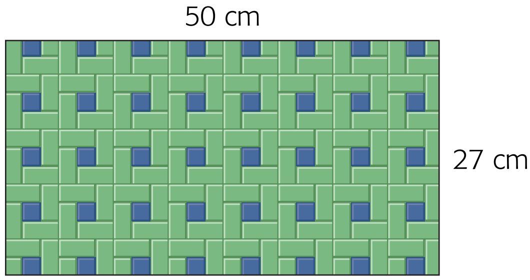
   b) Create a net of each prism. Check that they work by cutting them out and folding to create a model of a house.

7. Draw a net for a box that would just hold the tiles, stacked in one pile. Each box has to be the same shape as the tiles it holds.

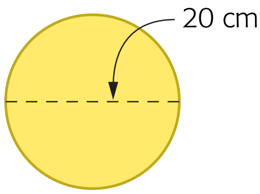
a) 30 triangular floor tiles 2 mm thick



b) 30 rectangular floor tiles 3 mm thick



c) 30 circles 2 cm thick



8. Jenna has 8 rolls of tape. Each roll is 40 cm in circumference and 7 cm high. Draw a net of a rectangular box that will fit all of the rolls in one layer.

9. a) Explain what strategies you can use to recognize whether a net is for a rectangular prism, a triangular prism, or a cylinder.

b) Explain what strategies you can use to draw a net for a rectangular prism, a triangular prism, and a cylinder. Draw an example of one of them.

Reading Strategy

Visualizing

Picture each net in your mind. Sketch what you think the nets will look like before using the measurements to create your drawings.

5.3 Determining the Surface Area of Prisms

YOU WILL NEED

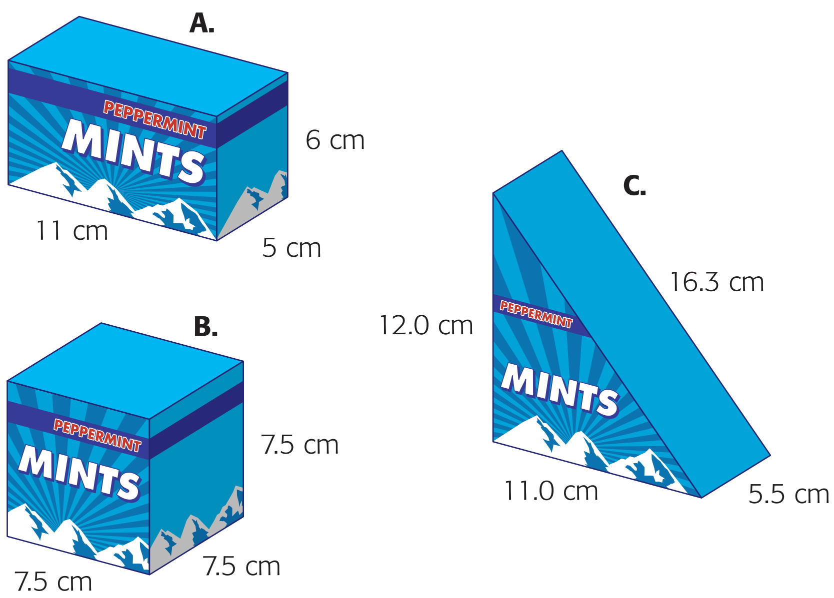
- 1 cm Grid Paper
- a calculator
- a ruler

GOAL

Develop strategies to calculate the surface area of prisms.

LEARN ABOUT the Math

The managers of a mint factory are choosing a box to hold breath mints. They will choose the box that uses the least amount of cardboard, including 10% more for overlap and folding.

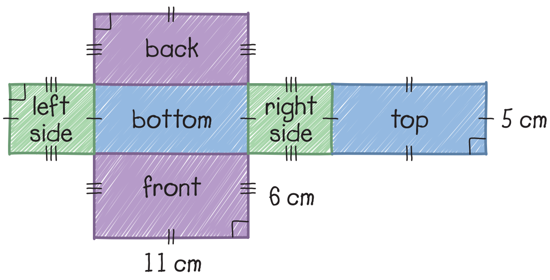


? Which box should be chosen?

Example 1 | Determining a rectangular prism's surface area

I determined the surface area (SA) of box A using a net.

Aaron's Solution



I imagined laying the box flat. I drew the net of the box and labelled the faces. Each face is a rectangle.

Area of front = 11 cm x 6 cm
= 66 cm^2
Area of back = 11 cm x 6 cm
= 66 cm^2
Area of right side = 5 cm x 6 cm
= 30 cm^2
Area of left side= 5 cm x 6 cm
= 30 cm^2
Area of top = 11 x 5 cm
= 55 cm^2
Area of bottom = 11 cm x 5 cm
= 55 cm^2

I calculated the area of each face.

SA = front + back + right side + left side + top + bottom
= 66 cm^2 + 66 cm^2 + 30 cm^2 + 30 cm^2 + 55 cm^2 + 55 cm^2
= 2(66 cm^2) + 2(30 cm^2) + 2(55 cm^2)
= 302 cm^2

To determine the surface area, I added all the areas. I noticed that the front and back had the same area. So did the sides, and so did the top and the bottom.

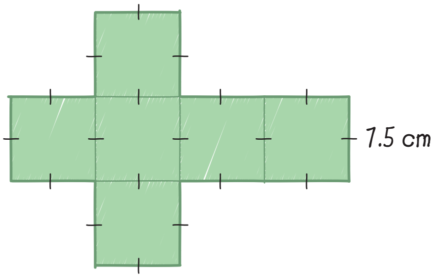
302 cm^2 x 0.10 = 30 cm^2
Total area of cardboard
= 302 cm^2 + 30 cm^2
= 332 cm^2
Box A uses 332 cm^2 of cardboard.

They want 10% more for overlap, so I calculated 10% and added it to the surface area.

Example 2 | Determining a cube's surface area

I determined the surface area of box B by recognizing that all of the faces are congruent.

Nikita's Solution



I drew the net. I noticed each face was 7.5 cm by 7.5 cm. The faces were congruent.

SA = 6 x area of one face
= 6×7.5 cm $\times 7.5$ cm
= 337.5 cm^2

I multiplied the area of one face by the number of faces.

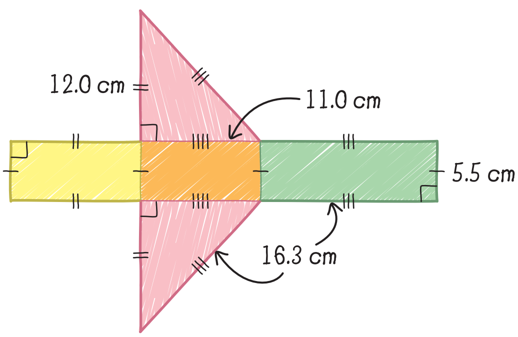
Total area of cardboard
= 337.5 cm^2 + 33.8 cm^2
= 371.3 cm^2
Box B uses 371.3 cm^2 of cardboard.

I added 10% to the surface area.

Example 3 | Determining a triangular prism's surface area

I determined the surface area of box C using the formula for the area of a triangle.

Brian's Solution



I drew the net.

Area of one triangle
= (b x h) ÷ 2
= (11.0 cm x 12.0 cm) ÷ 2
= 66.0 cm^2
Area of two triangles = 2 x 66.0 cm^2
= 132.0 cm^2

I calculated the area of the two triangles.

Area of rectangles
= 12.0 cm x 5.5 cm + 11.0 cm x 5.5 cm + 16.3 cm x 5.5 cm
= 66.0 cm^2 + 60.5 cm^2 + 89.7 cm^2
= 216.2 cm^2

I calculated the area of the rectangles.

SA = 132.0 cm^2 + 216.2 cm^2
= 348.2 cm^2

The surface area is the sum of the areas of the triangles and rectangles.

Total area of cardboard
= 348.2 cm^2 + 34.8 cm^2
= 383.0 cm^2
Box C uses 383.0 cm^2 of cardboard.

I added 10% to the surface area.

Box A uses 332 cm^2, box B uses 371.3 cm^2, and box C uses 383.0 cm^2 of cardboard. Box A uses the least cardboard.

Reflecting

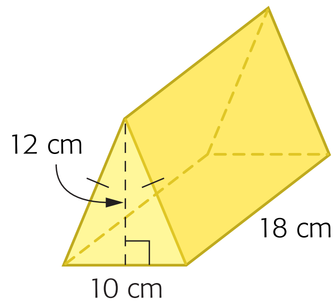
A. How does drawing the net of a prism help you calculate its surface area?

B. Why did Nikita's calculation require fewer steps than Brian's or Aaron's?

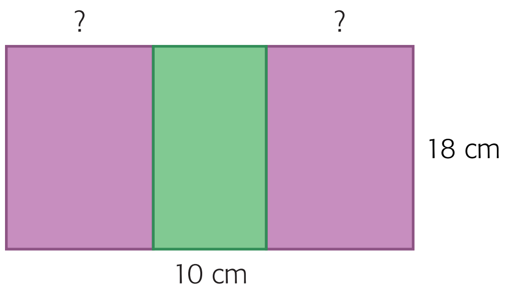
WORK WITH the Math

Example 4 | Calculating a triangular prism's surface area

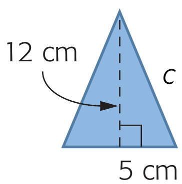
Calculate the surface area of this prism.



Solution

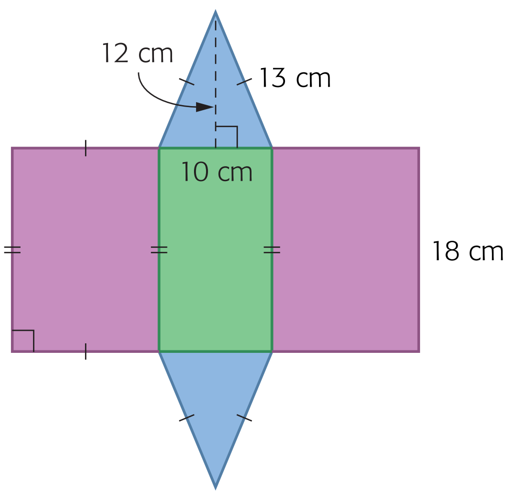


First sketch the part of the net that shows the rectangular faces. The widths of two rectangles are unknown.



$$c^2 = (5\text{ cm})^2 + (12\text{ cm})^2$$
$$= 169\text{ cm}^2$$
$$c = 13\text{ cm}$$

The widths were sides of the triangular base. Use the Pythagorean theorem to figure out what they are.



Draw the full net.

Area of two triangles
$= 2 \times (12\text{ cm} \times 10\text{ cm}) \div 2$
$= 120\text{ cm}^2$

Calculate the area of the faces. Multiply the areas of congruent faces.

Area of two side rectangles
$= 2 \times (18\text{ cm} \times 13\text{ cm})$
$= 468\text{ cm}^2$

Area of base rectangle
$= 18\text{ cm} \times 10\text{ cm}$
$= 180\text{ cm}^2$

$SA = 120\text{ cm}^2 + 468\text{ cm}^2 + 180\text{ cm}^2$
$= 768\text{ cm}^2$

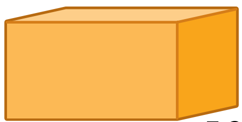
The surface area is the sum of all areas.

The surface area is 768 cm^2.

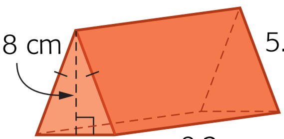
A Checking

1. Draw a net for each prism.

A.



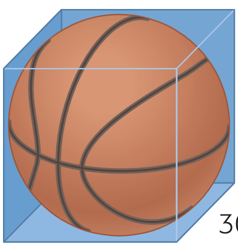
B.



2. Calculate the surface area of each prism in question 1.

B Practising

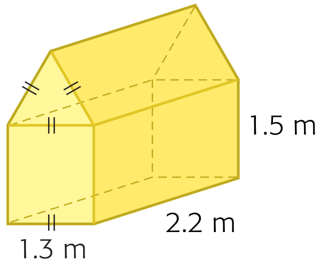
3. a) Sketch a rectangular prism 3 cm by 5 cm by 6 cm.
b) What is the surface area of the prism?



4. A sports company packages its basketballs in boxes. The boxes are shipped in wooden crates. Each crate holds 24 boxes.
a) Model three possible crates. Use centimetre cubes.
b) Draw nets for the three crates you modelled.
c) Calculate the surface area of each crate you modelled.
d) Which crate uses the least amount of wood?

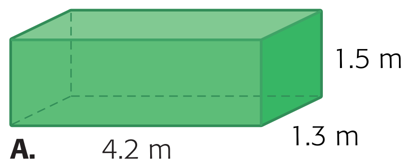
5. Marilynn has 1 m^2 of paper to wrap a box 28 cm long, 24 cm wide, and 12 cm high for a present. Does she have enough paper?

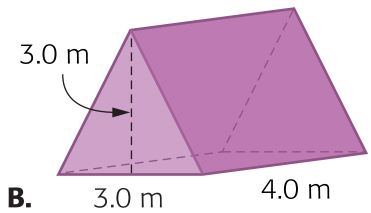
6. Alan is painting the walls and ceiling of his room, which is 4.2 m long, 3.7 m wide, and 2.6 m high. The window is 60 cm long by 40 cm high. The door is 2 m high by 85 cm wide.
   a) Determine the surface area of the walls in the room.
   b) He will use two coats of paint. A 4 L can of paint can cover 36 m^2. How many cans of paint does he need to buy?



7. Jordan is building this doghouse. (He will cut the door in the doghouse later.) How much wood will he need?

8. Which object has the greater surface area? Explain how you know.





9. Adrian cuts a cube into smaller cubes. Is the total surface area of the smaller cubes less than, greater than, or equal to the surface area of the original cube? Explain your thinking with words, diagrams, and calculations.

10. **a)** Draw a rectangular prism with a surface area of 24 cm^2.
    b) Draw a new rectangular prism where the sides are twice as long as the original. How does its surface area compare with that of the original?
    c) Draw a new rectangular prism where the sides are half as long as the original. How does its surface area compare with that of the original?

11. **a)** Calculate the surface area of a rectangular prism 10 m long, 8 m wide, and 6 m high.
    b) What might be the dimensions of a triangular prism with the same height and surface area as the prism in part a)?

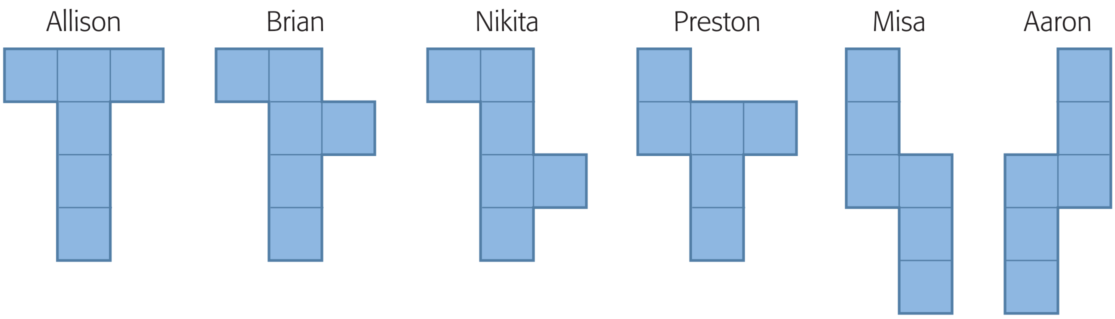
12. Why might you need to calculate the surface area of a prism?

13. **a)** How many areas would you add to calculate the surface area of a triangular prism? Explain.
    b) How many areas would you add to calculate the surface area of a rectangular prism? Explain.

CURIOUS Math

More than One Way to Net a Cube

Some students were asked to draw a net of a cube. This is what they drew.



Allison said, "We all drew different nets."

Brian said, "Misa's net and Aaron's net are really the same, though. They are just reversed."

Preston said, "All of our nets are correct. I wonder if there are other nets we could draw."

1. There are other nets of cubes. How many can you discover?
2. How many nets can you draw for a box in the shape of a cube that has no lid?

5.4 Determining the Surface Area of Cylinders

YOU WILL NEED

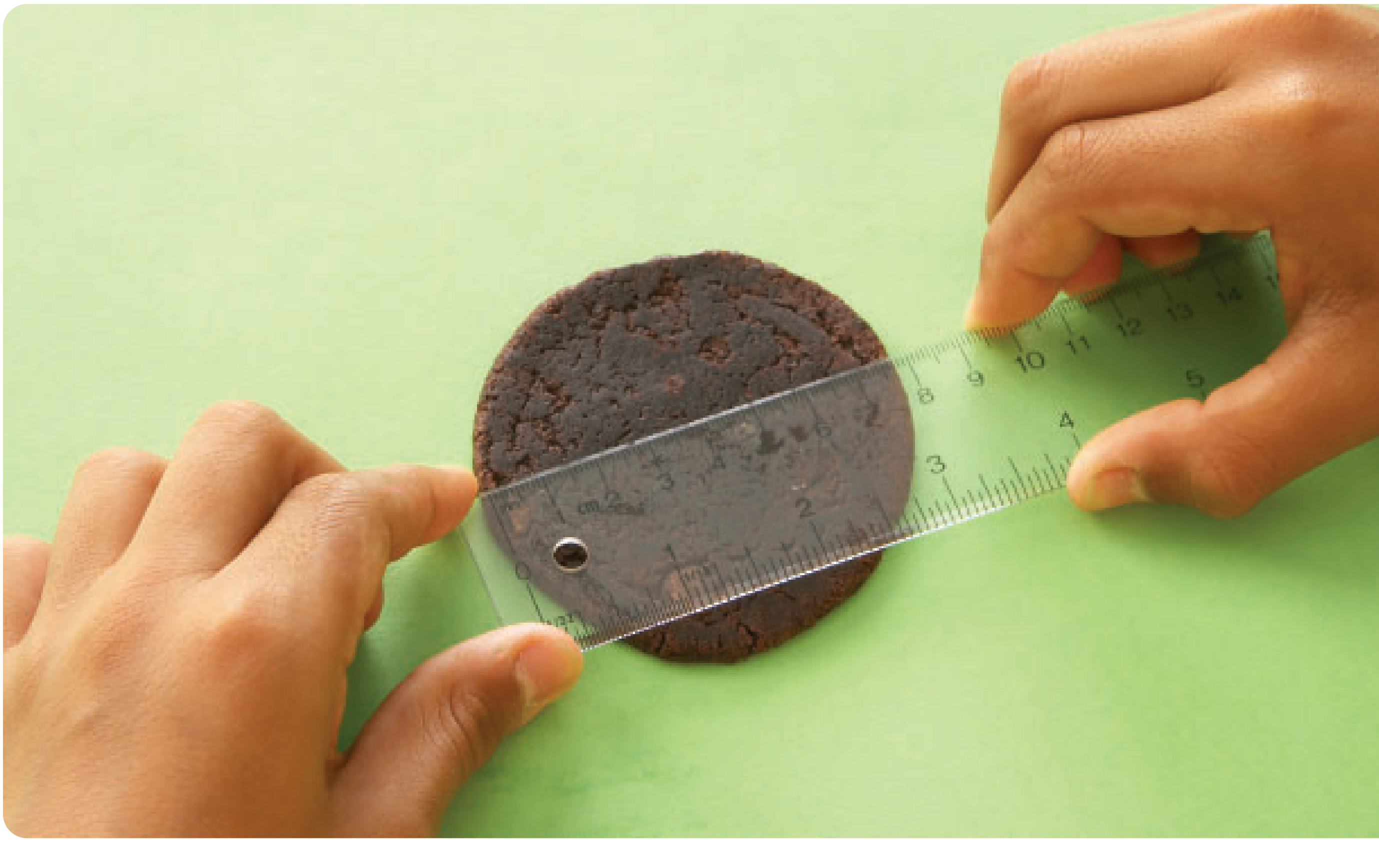
- 1 cm Grid Paper
- a calculator
- a ruler
- a compass

GOAL

Develop strategies to calculate the surface area of a cylinder.

LEARN ABOUT the Math

Preston and Misa are making cardboard packages for cookies for a school fundraiser. Each package will hold 12 cookies. They decide to add 5% additional cardboard for overlap.



How much cardboard do they need for each package?

A. Draw a net of the package.

B. Label the height of the package.

C. What is the area of the top of the package?
What is the area of the bottom of the package?

D. What is the area of the curved part of the package?

E. What is the surface area of the whole package?

F. What area of cardboard is needed for the package?

Reflecting

G. Which surface of a cylinder is affected by the cylinder's height?

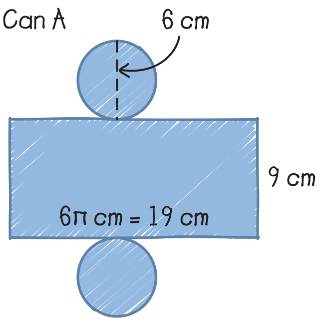
H. Write a formula for the surface area of a cylinder.

WORK WITH the Math

Example 1 | Estimating the surface area of a cylinder

Can A is 6 cm in diameter and 9 cm high.
Can B is 5 cm in radius and 4 cm high.
Which can has the greater surface area?

Aaron's Solution



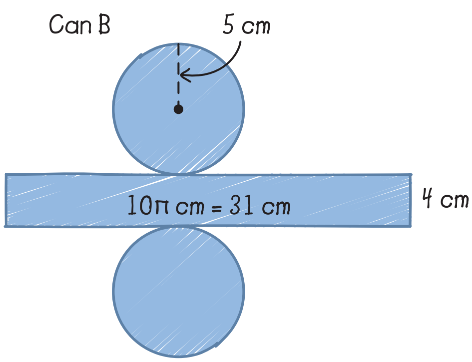
I drew the nets. The curved side became a rectangle. The width of the rectangle is the circumference of the circular base. For can A, the diameter is 6 cm, so the rectangle is about 3.14×6 cm $= 19$ cm wide.

The area of the rectangle is about
$10 \text{ cm} \times 20 \text{ cm} = 200 \text{ cm}^2$.
The area of each circle is about
$3 \times 3 \text{ cm} \times 3 \text{ cm} = 27 \text{ cm}^2$. ($SA = \pi r^2$)

I estimated using easier numbers.

$SA \doteq 27 \text{ cm}^2 + 27 \text{ cm}^2 + 200 \text{ cm}^2$
$\doteq 254 \text{ cm}^2$
Can A has a surface area of about 254 cm^2.

I added all the areas.



For can B, the diameter was 10 cm, so the rectangle was about $3 \times 10 = 30$ cm wide.

The area of the rectangle is about
$4 \text{ cm} \times 30 \text{ cm} = 120 \text{ cm}^2$.
The area of each circle is about
$3 \times 5 \text{ cm} \times 5 \text{ cm} = 75 \text{ cm}^2$.

$SA \doteq 75 \text{ cm}^2 + 75 \text{ cm}^2 + 120 \text{ cm}^2$
$\doteq 270 \text{ cm}^2$
Can B has a surface area of about 270 cm^2.
Can B has the greater surface area.

I added all the areas.

Example 2 | Calculating the surface area of a cylinder

Allison is wrapping a cylindrical candle 7.5 cm high and 3.5 cm in diameter as a present for her mother. Allowing 5% for overlap, what area of wrapping paper does she need?

Allison's Solution

The radius is $3.5 \text{ cm} \div 2 = 1.8 \text{ cm}$.

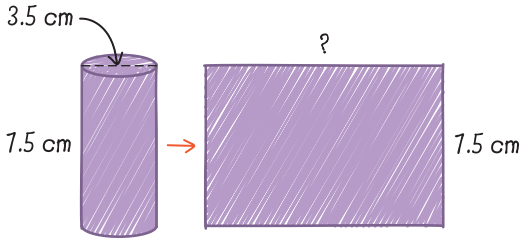
Area of top and base

$= 2 \times \pi \times r \times r$

$\doteq 2 \times 3.14 \times 1.8 \text{ cm} \times 1.8 \text{ cm}$

$\doteq 20.3 \text{ cm}^2$

The top and the base have the same area. I determined the area of one face and doubled it. I decided not to estimate using 3 instead of 3.14 because my estimate might come out too low.



When you unroll the curved side of the candle, it forms a rectangle. The sides of the rectangle are the circumference of the base and the height of the cylinder. The circumference of the base is πd.

$\pi d = \pi \times 3.5 \text{ cm}$

$\doteq 10.99 \text{ cm}$

$\doteq 11.0 \text{ cm}$

Area of curved surface $= C \times h$

$= \pi d \times h$

$\pi d \times h \doteq 11.0 \text{ cm} \times 7.5 \text{ cm}$

$\doteq 82.5 \text{ cm}^2$

$SA \doteq 20.3 \text{ cm}^2 + 82.5 \text{ cm}^2$

$\doteq 102.8 \text{ cm}^2$

The surface area is the sum of all the areas.

Total area of paper

$\doteq 102.8 \text{ cm}^2 + 5.1 \text{ cm}^2$

$\doteq 107.9 \text{ cm}^2$

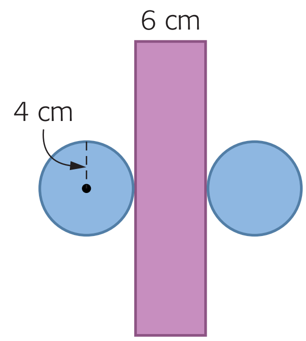
I need about 108 cm^2 of paper.

I added 5% to determine the total area.

A Checking

1. Determine the surface area of each cylinder, using the net.

a)



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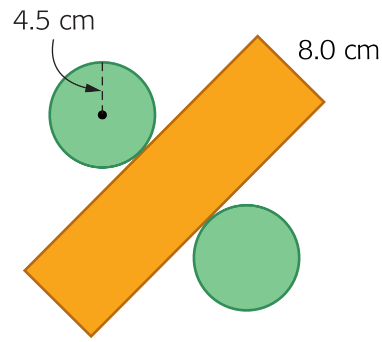
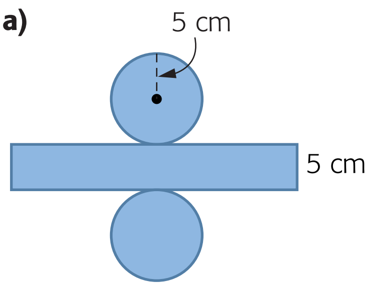


B Practising

2. Calculate the surface area of each cylinder.

a)



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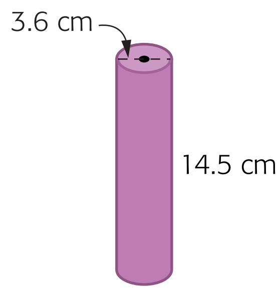
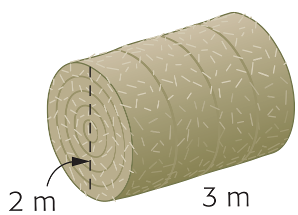


3. Determine the surface area of each cylinder.

	Diameter (cm)	Height (cm)
a)	10.0	8.0
b)	10.0	6.5
c)	10.0	9.4

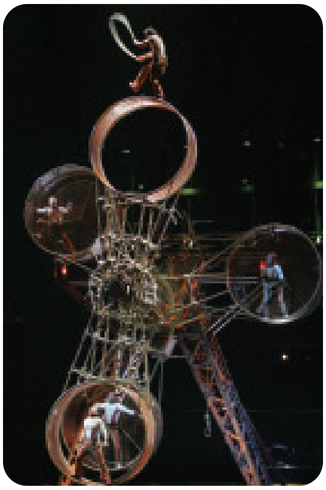
4. A farmer is buying wrap to protect her hay bales. Each bale is 2 m in diameter and is 3 m high. The top and the bottom of the bales are not enclosed. How much wrap does each bale require?



5. A can of frozen juice that is 6.7 cm in diameter and 11.8 cm high is made of a cardboard tube, and a metal top and metal bottom. Suppose 24 cans are recycled.
   a) Determine how much cardboard is recycled.
   b) Determine how much metal is recycled.

6. a) This railway car is 3.2 m in diameter and 17.2 m long. Calculate its surface area.
   b) A can of paint covers 40 m^2 and costs \$35. Estimate the cost to paint the railway car.

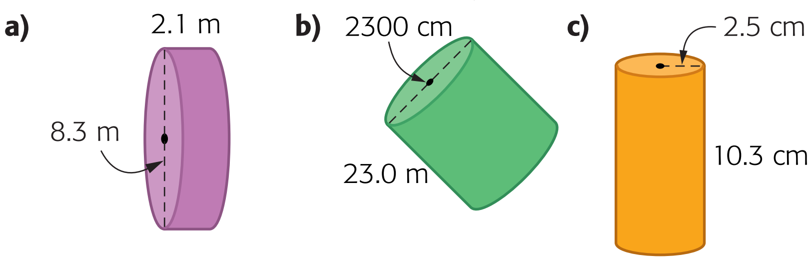
7. Explain how two cylinders can have the same height but different surface areas.



8. This acrobatic stunt is from the Cirque de Soleil. Each wheel is about 30 cm wide and 2.5 m in diameter. What is the surface area of each wheel?

9. Brian is buying burlap to protect his three apple trees against winter weather. He will wrap the burlap around the bottom 150 cm of each tree trunk. The trees are 25.1 cm, 29.8 cm, and 31.4 cm in circumference. About how much burlap will he need?

10. Calculate the surface area of each cylinder.



11. A cylindrical CD case has a surface area of 372.0 cm^2. Each CD is 0.1 cm thick and 11.0 cm in diameter. How many CDs can the case hold? Explain, with the help of formulas, what you did.

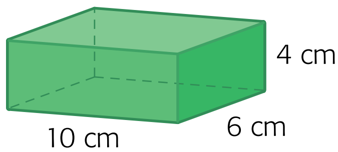
12. How are calculating the surface area of a cylinder and calculating the surface area of a prism alike? How are they different?

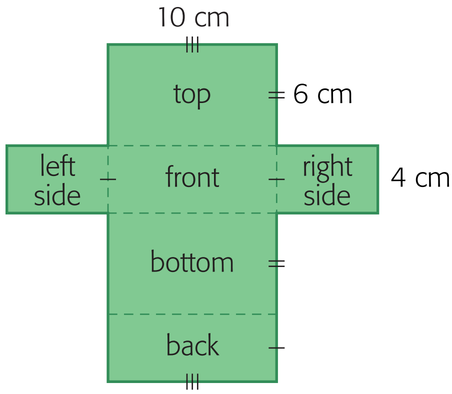
Mid-Chapter Review

Frequently Asked Questions

Q: How do you calculate the surface area of a prism?

A: The surface area is the sum of the areas of the faces. For a rectangular prism, three pairs of faces are congruent: the front and back, the left and right sides, and the top and bottom. So calculate the area of one face in each pair and double that. Add to determine the total area.

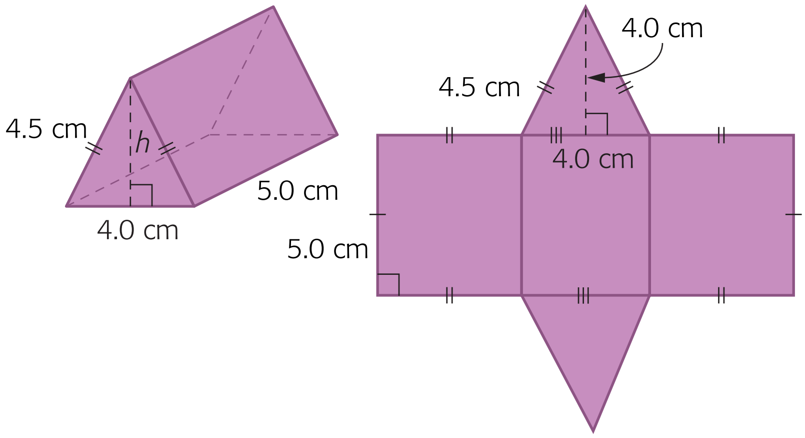




$$\begin{aligned}\text{Surface area} &= 2 \times \text{area of top} + 2 \times \text{area of front} \\ &\quad + 2 \times \text{area of left side} \\ &= 2(6\text{ cm} \times 10\text{ cm}) + 2(4\text{ cm} \times 10\text{ cm}) \\ &\quad + 2(4\text{ cm} \times 6\text{ cm}) \\ &= 120\text{ cm}^2 + 80\text{ cm}^2 + 48\text{ cm}^2 \\ &= 248\text{ cm}^2\end{aligned}$$

For a triangular prism, two of its five faces, the triangular bases, are congruent. The other three faces may or may not be congruent. To calculate the area of the bases, you may need to determine their height.

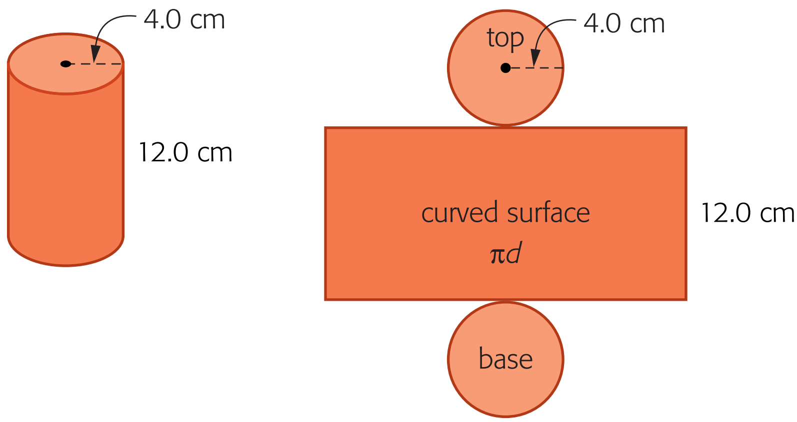
By the Pythagorean theorem, each base of the following prism has a height of 4.0 cm.



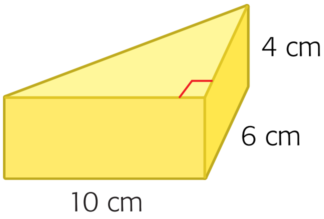
$$\begin{aligned}\text{Surface area} &= 2 \times \text{area of bases} + 2 \times \text{area of sides} \\ &\quad + \text{area of bottom} \\ &= 2(4.0\text{ cm} \times 4.0\text{ cm} \div 2) + 2(4.5\text{ cm} \times 5.0\text{ cm}) \\ &\quad + (4.0\text{ cm} \times 5.0\text{ cm}) \\ &= 16.0\text{ cm}^2 + 45.0\text{ cm}^2 + 20.0\text{ cm}^2 \\ &= 81.0\text{ cm}^2\end{aligned}$$

Q: How do you calculate the surface area of a cylinder?

A: You can draw a net, if you wish. The curved surface becomes a rectangle where length is the cylinder's circumference and width is the cylinder's height. The base and the top are congruent, so they have the same area.



$$\begin{aligned}\text{Surface area} &= 2(\text{area of base}) + \text{area of curved surface} \\ &= 2(\pi \times r \times r) + (\pi d \times h) \\ &\doteq 2(3.14 \times 4.0\text{ cm} \times 4.0\text{ cm}) \\ &\quad + (3.14 \times 8.0\text{ cm} \times 12.0\text{ cm}) \\ &\doteq 401.9\text{ cm}^2\end{aligned}$$

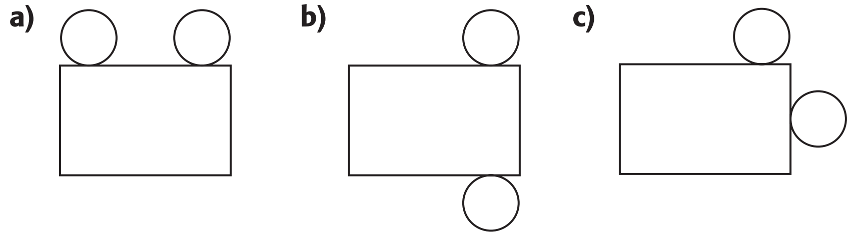


Practice

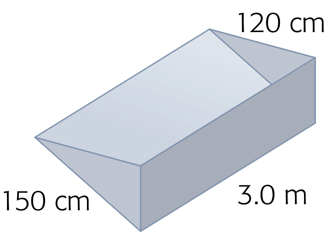
Lesson 5.2

1. Draw the net of this prism.

2. State whether each net will fold to make a soup can. If it will not, explain why.



Lesson 5.3



3. Megan is painting a rectangular box 18 cm by 5 cm by 2 cm. What surface area does she need to paint?

4. Emma's dad is building a triangular hay trough for his horses, as shown. How much wood will he need?

Lesson 5.4

5. Sketch a net for each cylinder, and label its dimensions. Then calculate the surface area.

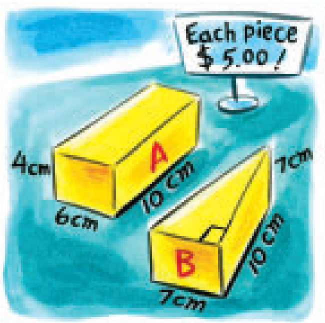
	Item	Radius (cm)	Height (cm)
a)	potato-chip container	4	8
b)	coffee can	7.5	15.0
c)	CD case	8.5	20.5
d)	oil barrel	25.0	80.0

6. Karim is painting a barrel 1.2 m high and 0.3 m in radius. Including the top and bottom, what area will the paint have to cover?

5.5 Determining the Volume of Prisms

GOAL

Develop and apply formulas for the volume of prisms.



LEARN ABOUT the Math

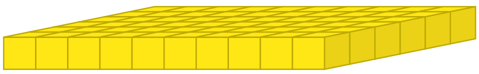
Misa wants to buy a piece of cheese.

? Which piece of cheese is the better buy?

Example 1 | Calculating the volume of a rectangular prism

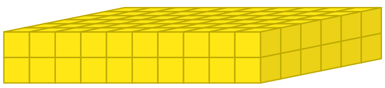
I used a model to calculate the volume of piece A.

Misa's Solution



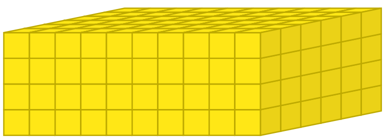
This prism has 60 cubes, so its volume is 60 cm^3.

I modelled one layer with centimetre cubes. The area of the base was 60 cm^2 and the height was 1 cm.



This prism has 120 cubes, so its volume is 120 cm^3.

For two layers, the area of the base was 60 cm^2 and the height was 2 cm.



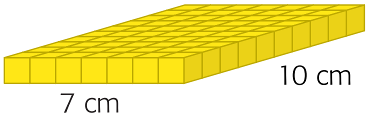
This prism has 240 cubes, so piece A has a volume of 240 cm^3.

For four layers, the area of the base would be 60 cm^2 and the height 4 cm. I thought the volume would be 240 cm^3. I was right.

Example 2 | Calculating the volume of a triangular prism

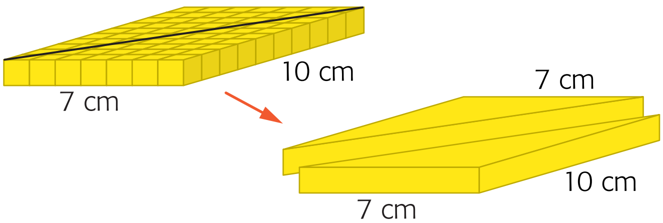
I imagined a model to calculate the volume of piece B.

Brian's Solution



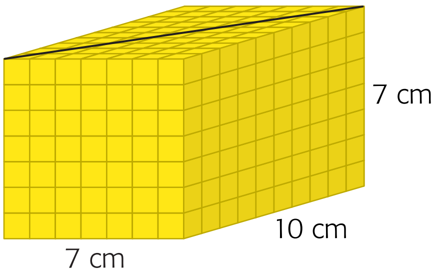
This prism has 70 cubes, so its volume is 70 cm^3.

I modelled a rectangular prism 1 cm high with the same length and width as piece B.



I imagined cutting it along the diagonal to form two congruent triangular prisms. Each piece would have half the volume of the original prism.

Volume of one layer = 70 cm^3 ÷ 2, or 35 cm^3



This prism has 490 cubes, so its volume is 490 cm^3.

I modelled a rectangular prism with the same width, length, and height as piece B.

Piece B has half the volume of this prism.

Volume of B = 490 cm^3 ÷ 2, or 245 cm^3

Piece B is the better buy.

Reflecting

A. Write a formula for the volume of a rectangular prism.

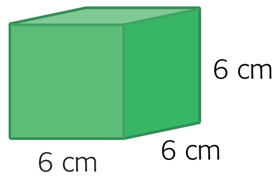
B. Is every triangle half of a rectangle?

C. Write a formula for the volume of a triangular prism.

WORK WITH the Math

Example 3 | Calculating the volume of a rectangular prism

Calculate the volume of this prism.



Solution

Area of base = length × width

$B = l \times w$
$= 6 \text{ cm} \times 6 \text{ cm}$
$= 36 \text{ cm}^2$

Calculate the area of the base.

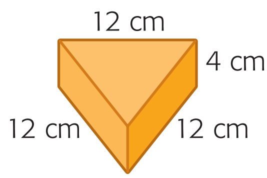
Volume $= B \times h$
$= 36 \text{ cm}^2 \times 6 \text{ cm}$
$= 216 \text{ cm}^3$

Multiply the area of the base by the height.

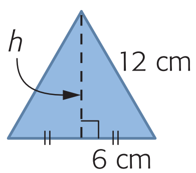
This prism has a volume of 216 cm^3.

Example 4 | Calculating the volume of a triangular prism

Calculate the volume of this prism.



Solution



$h^2 = (12 \text{ cm})^2 - (6 \text{ cm})^2$
$= 144 \text{ cm}^2 - 36 \text{ cm}^2$
$= 108 \text{ cm}^2$
$h \doteq 10 \text{ cm}$

Calculate the height of the base. Use the Pythagorean theorem.

$A = B \times h \div 2$
$= 12 \text{ cm} \times 10 \text{ cm} \div 2$
$= 60 \text{ cm}^2$

Determine the area of the base.

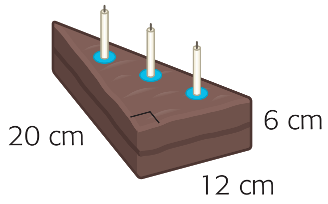
$V = B \times h$
$= 60 \text{ cm}^2 \times 4 \text{ cm}$
$= 240 \text{ cm}^3$

Multiply the area of the base by the height of the prism.

This prism has a volume of about 240 cm^3.

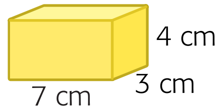
Communication Tip

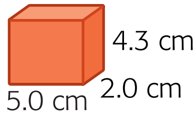
In a formula, *h* can refer to the height of a triangle, or to the height of a prism. Take care to use the appropriate value.

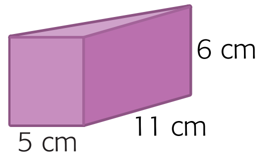


A Checking

1. Calculate the volume of each prism.

a) 

b) 

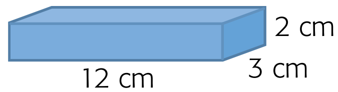
c) 

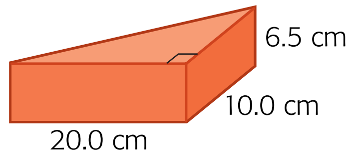
B Practising

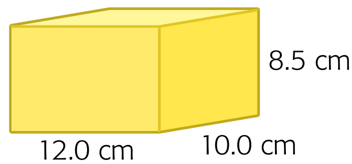
2. a) This slice is half the volume of a rectangular cake. What was the volume of the whole cake?

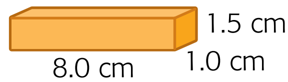
b) Calculate the volume of this slice of cake.

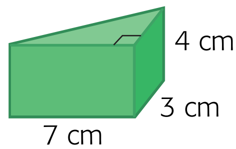
3. Calculate the volume of each prism.

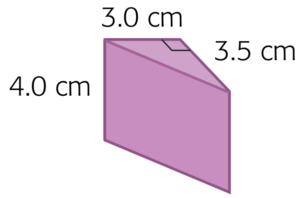
a) 

b) 

c) 

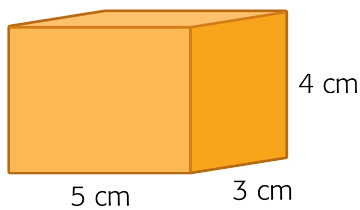
d) 

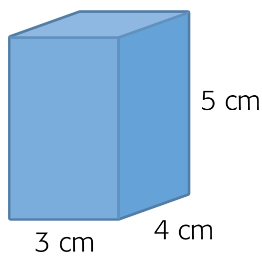
e) 

f) 

4. a) Determine the volume of prism A.

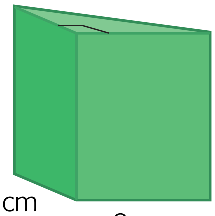
b) Do you need to calculate to determine the volume of prism B? Explain.

A. 

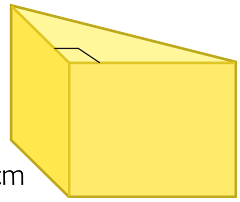
B. 

5. a) Determine the volume of prism A.
   b) Do you need to calculate to determine the volume of prism B? Explain.

A.



B.



6. Sketch a rectangular prism with each set of dimensions and then calculate its volume.
   a) $l = 8$ cm, $w = 8$ cm, $h = 8$ cm
   b) $l = 0.5$ cm, $w = 0.5$ cm, $h = 2.0$ cm
   c) $l = 3.5$ km, $w = 2.0$ km, $h = 3.0$ km

7. Copy and complete the table for rectangular prisms.

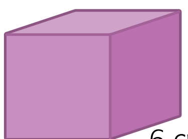
	Length (cm)	Width (cm)	Height (cm)	Volume (cm^3)
a)	6	6	8	
b)	4.5	5.0		216.0
c)	3		3	27

8. Copy and complete the table for triangular prisms.

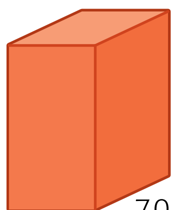
	Length (cm)	Width of Base (cm)	Height of Base (cm)	Volume (cm^3)
a)	6	6	8	
b)	3	5		300

9. Anthony needs to buy nails for his carpentry project. The hardware store sells these boxes of nails for the same price. Which one should he buy? Explain your choice with a sketch, calculations, and words.

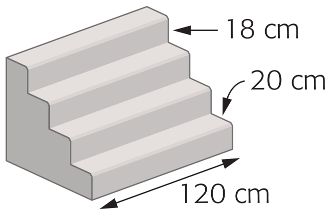
A.



B.



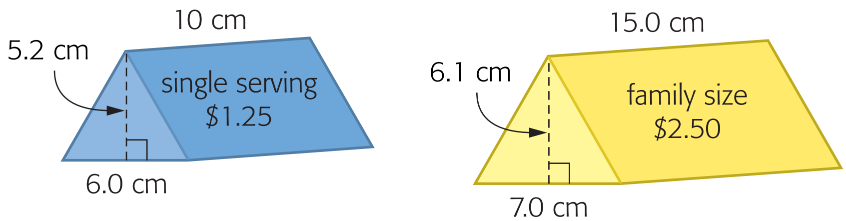
10. Samantha has to pack 30 books in a box. Twenty books are each 28 cm by 21 cm by 2 cm. Ten books are each 20 cm by 18 cm by 3 cm. What is the least volume the box can have?



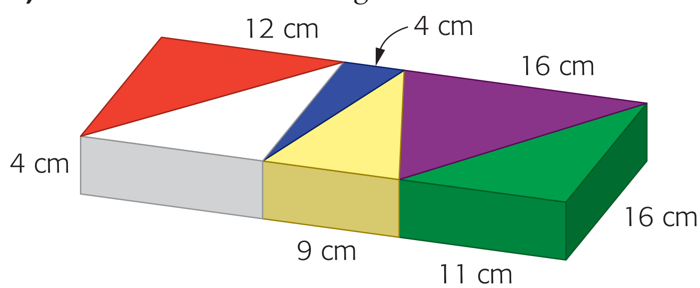
11. The concrete steps to Brian's front door are shown. What volume of cement was needed to build the steps?

12. **a)** Draw a rectangular prism with a volume of 24 cm^3.
    b) Draw a new rectangular prism where the sides are twice as long as the original. How does its volume compare with that of the original?
    c) Draw a new rectangular prism where the sides are half as long as the original. How does its volume compare with that of the original?

13. Raisins are sold in two different boxes. Which one do you think is better in terms of getting more raisins for your money?



14. Allan's teacher bought solid water colour cakes in a tray, as shown.
    a) Determine the volume of each colour.
    b) Which colour had the greatest volume?



15. Estimate the volume of space in your classroom.

16. Will a rectangular prism and a triangular prism have the same volume if they are both the same height? Explain.

5.6 Determining the Volume of Cylinders

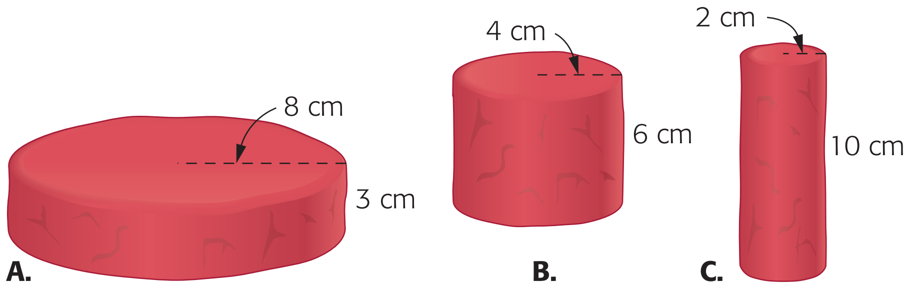
YOU WILL NEED
- 1 cm Grid Paper
- a compass
- centimetre cubes
- a calculator

GOAL

Develop a formula for the volume of a cylinder.

LEARN the Math

Allison is going to buy some modelling clay. Each cylinder costs $5.



? Which choice is the best buy?

A. Draw a circle with the same radius as the base of cylinder A. Estimate its area.

B. Stack centimetre cubes to model the height of cylinder A.

C. Estimate the volume of cylinder A.

D. Repeat steps A to C for the other two cylinders.

E. Which choice is the best buy? Explain.

Reflecting

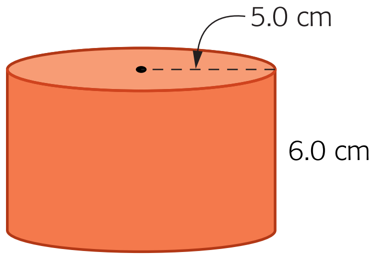
F. How can you estimate a cylinder's volume using its radius and height?

G. Use the formula for the volume of a rectangular prism to create a formula for the volume of a cylinder. Explain your thinking.

***WORK WITH** the Math*

Example 1 | Calculating the volume of a cylinder

Calculate the volume of this cylinder.



Allison's Solution

Volume of cylinder
= area of base x height
= π x r x r x height
≐ 3.14 x 5.0 cm x 5.0 cm x 6.0 cm
≐ 471.0 cm³
The volume is about 471.0 cm³.

I calculated the volume the way I would calculate the volume of a prism: I multiplied the area of the base by the height.

Example 2 | Using volume to solve a problem

A tube of cookie dough is 942 cm^3 in volume and 10 cm in diameter. Each cookie will be 1 cm thick. How many cookies can Nikita make?

Nikita's Solution

Volume of one cookie
= area of base x height
= π x r x r x h
≐ 3.14 x 5.0 cm x 5.0 cm x 1 cm
≐ 78.5 cm³

I calculated the volume of one cookie.

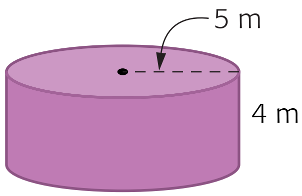
Total volume ÷ volume of one cookie
= 942 cm³ ÷ 78.5 cm³
= 12
I can make 12 cookies.

To determine how many cookies I could make, I divided the total volume of the cookie dough by the volume of one cookie.

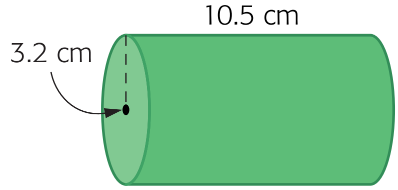
A Checking

1. Calculate the volume of each cylinder.

a)



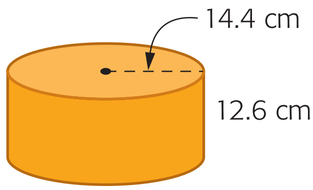
b)



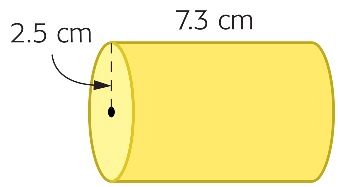
B Practising

2. Calculate the volume of each cylinder.

a)

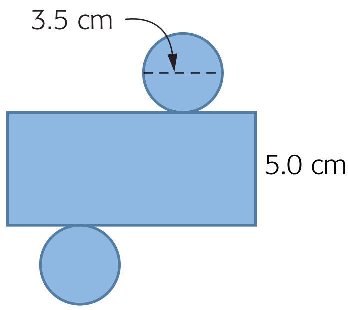


b)

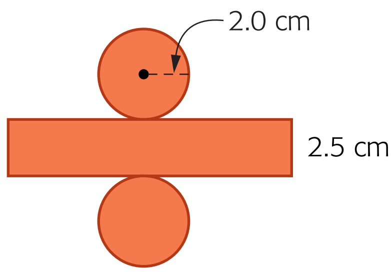


3. Determine the volume of the cylinder you could create with each net.

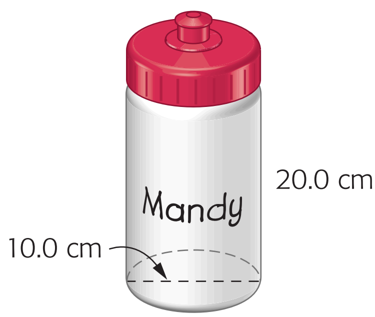
a)



b)



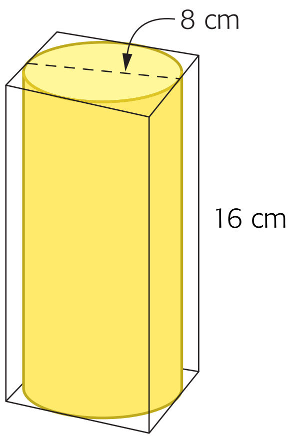
4. There are 12 people in Mandy's exercise class. Each one has a water bottle like this. They fill their bottles from a water cooler that is 20 cm in radius and 90 cm in height. Estimate how many times they can fill up their water bottles before the cooler needs to be refilled.



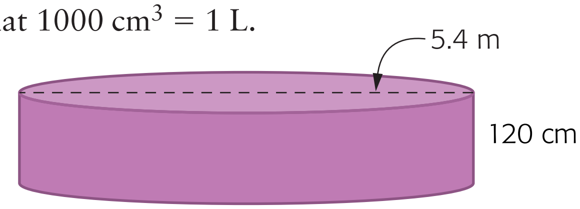
Reading Strategy

Summarizing

In your own words, how would you summarize the key idea in this lesson?
How does it apply to this problem?



5. Estimate the number of litres of water in this swimming pool. Recall that $1000\ cm^3 = 1$ L.

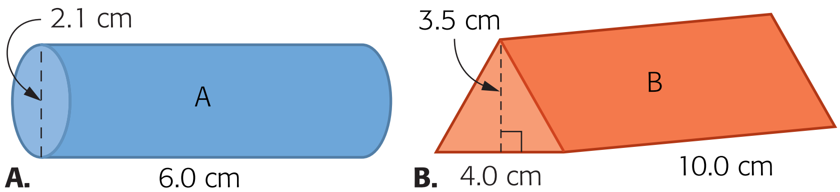


6. A cylindrical candle is sold in a gift box that is a square-based prism. Determine the volume of the empty space in the box.

7. Determine the height of a cylinder with a base area of $50.2\ cm^2$ and a volume of $502.4\ cm^3$.

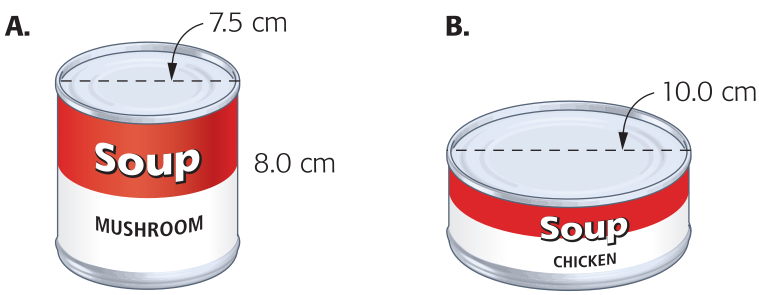
8. Loren is putting \$2 coins into a plastic tube to take to the bank. The tube has a volume of $26.9\ cm^3$. A \$2 coin is 1.75 mm thick and 28.00 mm in diameter. How many \$2 coins will the tube hold?

9. Which container holds more? Justify your answer.



10. Which holds more flour, a cylinder 10.0 cm high and 7.0 cm in diameter or a cylinder 7.0 cm high and 10.0 cm in diameter?

11. These two metal cans both hold the same amount of soup.
    a) Determine the height of the can of chicken soup. Show your solution.
    b) Which can uses more metal? Show your work.



12. How are calculating the volume of a prism and calculating the volume of a cylinder alike? How are they different?

5.7 Solve Problems Using Models

GOAL

Use models to solve measurement problems.

LEARN ABOUT the Math

Brian's mom has 8 m^3 of sand left over from a gardening project. She asked Brian to design a wooden sandbox, with a bottom and a top, for his little sister, Sally. Brian has decided that the sandbox should have these features.

- It should be 50 cm deep, so Sally can climb in and out easily but still have enough to dig in.
- It should use the least amount of wood to save money.
- Its base should be square or triangular.

? Which design should Brian choose?

Example 1 | Measuring rectangular and triangular prisms

I decided to use a model to solve the problem.

Brian's Solution

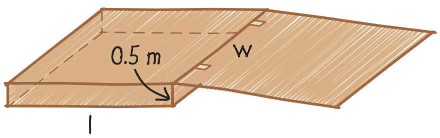
1. Understand the Problem

I assume the sand will fill the sandbox, so I will imagine each of my models is made of sand. Each model will be 0.5 m deep and contain 8 m^3 of sand.

2. Make a Plan

I know one dimension and the volume of each sandbox, so I can figure out the other dimensions.

3. Carry Out the Plan

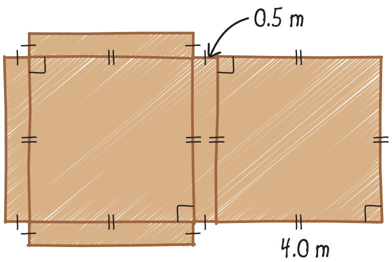


I used the lid of a greeting card box to represent the square sandbox. A square sheet of paper can represent the top.

I used the volume to figure out the other dimensions.

$$V = Bh$$
$$8\text{ m}^3 = l \times w \times 0.5\text{ m}$$
$$16\text{ m}^2 = l \times w$$

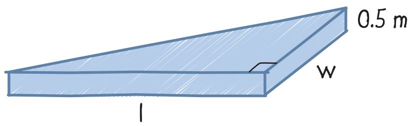
The box is a square, so $l = w = 4.0$ m.



I opened up the box lid to form the net of the sandbox. I determined the surface area of the lid and of the sheet of paper.

$$\begin{aligned} SA &= 2(4.0\text{ m} \times 4.0\text{ m}) + 4(4.0\text{ m} \times 0.5\text{ m}) \\ &= 32.0\text{ m}^2 + 8.0\text{ m}^2 \\ &= 40.0\text{ m}^2 \end{aligned}$$

There are two congruent squares and four congruent rectangles.



$V = (l \times w \div 2) \times h$

$8\ m^3 = (l \times w \div 2) \times 0.5\ m$

I used a plastic sandwich container to represent the triangular sandbox. I thought that the top of the container could represent the cover.
I used the volume to figure out the other dimensions.
I put the values I knew into the formula.

$16\ m^2 = l \times w \div 2$

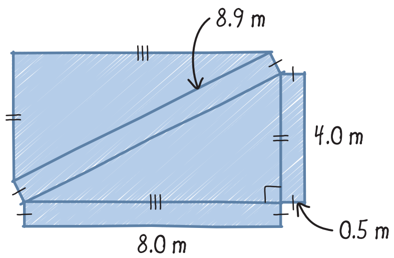
I divided both sides by 0.5. That is like multiplying by 2.

$32\ m^2 = l \times w$

I multiplied both sides by 2 again.

I chose 8.0 m for l and 4.0 m for w. By the Pythagorean theorem, the third side is 8.9 m.

I decided to choose 8.0 and 4.0 because they are factors of 32, and they are easy numbers to work with.



I cut the box to make the net and then I calculated the surface area.

$SA = 2[(8.0\ m \times 4.0\ m) \div 2] + 0.5\ m(4.0\ m + 8.0\ m + 8.9\ m)$
$= 32.0\ m^2 + 10.5\ m^2$
$= 42.5\ m^2$

We should build the square sandbox because it has less surface area and so it will use less wood.

4. Look Back

I checked my calculations. They look correct.
I thought that the sand would fill the box to the brim, but now I think it would be better not to fill the box to the top, so that the sand will not spill out.

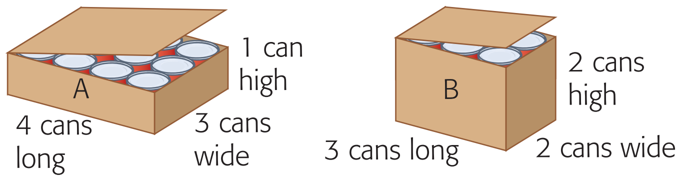
Reflecting

A. How did Brian's models help him figure out how much wood was needed to make the sandbox?

WORK WITH the Math

Example 2 | Solving a problem using models

A soup can has a capacity of 350 mL and radius of 3.0 cm. Which box uses less cardboard?



Nikita's Solution

1. Understand the Problem

Each can is 3.0 cm in radius and holds 350 mL of soup.
I will assume that each can has a volume of 350 cm^3.

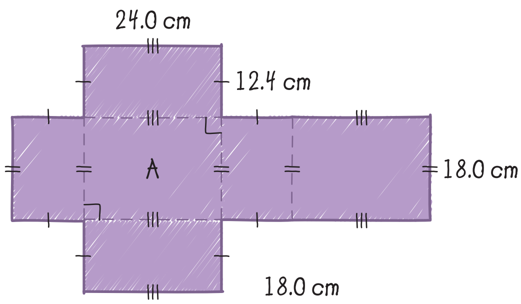
2. Make a Plan

I will use the volume to figure out the height of each can.
Then I will determine the surface areas of the boxes.

3. Carry Out the Plan

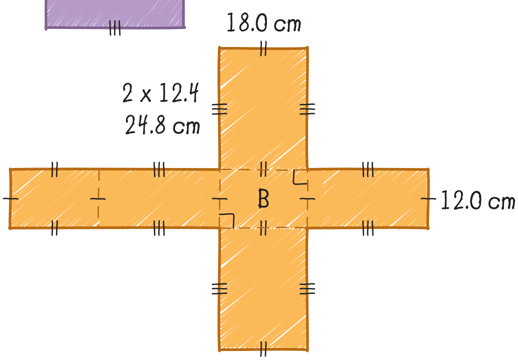
$$V = \pi \times r \times r \times h$$
$$350.0 \text{ cm}^3 \doteq 3.14 \times 3.0 \text{ cm} \times 3.0 \text{ cm} \times h$$
$$12.4 \text{ cm} \doteq h$$

I calculated the height of a can.



I determined the dimensions of each box.

$$SA = 2(18.0 \text{ cm} \times 24.0 \text{ cm}) + 2(12.4 \text{ cm} \times 24.0 \text{ cm}) + 2(12.4 \text{ cm} \times 18.0 \text{ cm})$$
$$= 1905.6 \text{ cm}^2$$

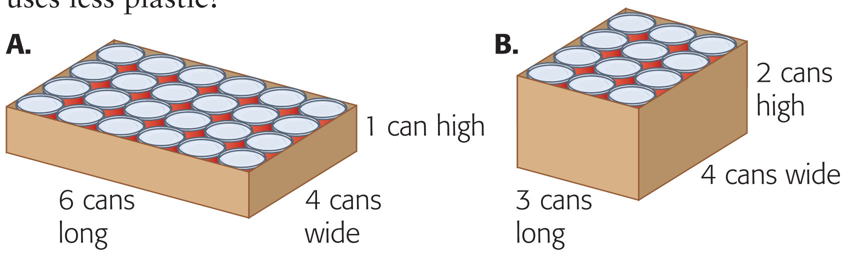


$$SA = 2(12.0 \text{ cm} \times 24.8 \text{ cm}) + 2(18.0 \text{ cm} \times 24.8 \text{ cm}) + 2(12.0 \text{ cm} \times 18.0 \text{ cm})$$
$$= 1920.0 \text{ cm}^2$$

Box A has less surface area, so it uses less cardboard.

A Checking

1. A can of vegetable juice has a capacity of 284 mL and is 3.2 cm in radius. Twenty-four cans of juice will be packed in open boxes, and then wrapped in plastic. Which arrangement uses less plastic?



B Practising

2. Circular tea bags are packaged in cylinders 8.0 cm high that are also 400 cm^3 in total volume. The cylinders are packed in boxes for shipping.
   a) Draw a model for two different boxes that would each hold 24 tea cylinders.
   b) In which box would you ship the tea cylinders? Explain.

3. Fritz is making a stained-glass window. This window is shaped like a rectangle 0.5 m wide by 2.5 m long, with a semicircle above the rectangle.
   a) Draw an outline of the window. Label the dimensions.
   b) How much glass does Fritz need?

4. A package of rice crackers is in the shape of a prism with a base area of 18.0 cm^2 and a volume of 216 cm^3. The base is a right isosceles triangle. The packages are shipped in boxes with a volume of 5184 cm^3.
   a) How many packages of crackers are in each box?
   b) Model two different boxes that would hold the packages. Explain which box you would use.

5. A pizza box measuring 34 cm by 34 cm by 5 cm contains a pizza that is 30 cm in diameter. About what percent of the box is occupied by the pizza and what percent is not?

6. Modelling is often a useful way to solve a problem. Is there a time when you would not use a model to solve a problem that involves surface area?

Math **GAME**

Matching Geometric Solids

YOU WILL NEED
- Geometric Solids Cards I–V
- a calculator

In this game, you will match cards of solids and their nets.

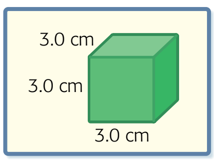
Number of players: 2–4

How to Play

1. Deal five cards to each player. Place the remaining cards in a pile on the table, face down.
2. In turn, put any matching pair of cards in front of you on the table. For example, you can match a net and a surface area, a 3-D object and a net, a 3-D object and a volume, or a 3-D object and a surface area. Then pick up two more cards from the pile.
3. If you cannot match any cards, ask another player for a matching card. If she has one, put the match on the table and take two more cards from the pile. If she does not have one, she says, “Go fish!” Then you pick one card from the pile. If you can make a match now, then do so, and take two more cards. If you cannot, then it is the next player’s turn.
4. If you disagree with a player’s match, make a challenge. If he is correct, he keeps the cards. If he is wrong, he gives you one of his matches.
5. The game is over when no one has any cards left.
6. The winner is the player who makes the most matches.

Nikita’s Turn

I had this card.



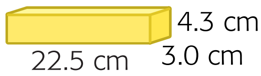
I could not match it with any of my other cards, so I asked Preston if he had one with $V = 27.0\ \text{cm}^3$ on it. He did not, so he said, “Go fish!” I took this card from the pile: so I had a match. I put down those two cards and took two more from the pile.

$SA = 54.0\ \text{cm}^2$

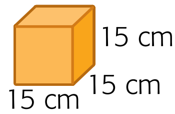
Chapter Self-Test

1. Calculate the surface area of each prism.

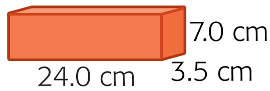
a)



b)

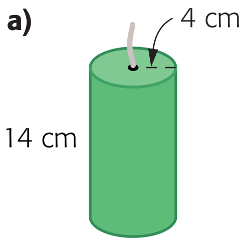


c)

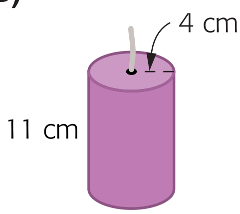


2. Draw a net for the paper that is needed to wrap each candle.

a)



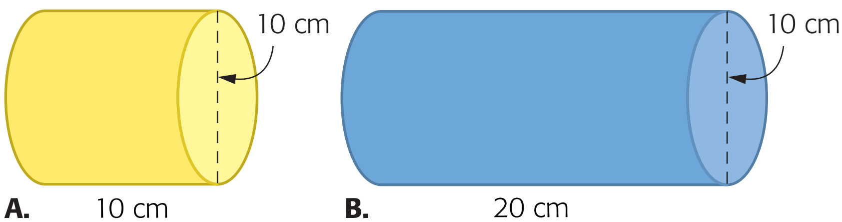
b)



3. Which one of the following two statements is true? Explain.

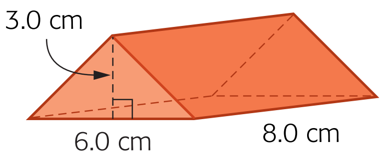
a) The volume of cylinder B is twice the volume of cylinder A.

b) The surface area of cylinder B is twice the surface area of cylinder A.

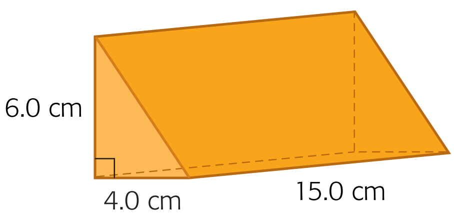


4. Calculate the surface area of each prism.

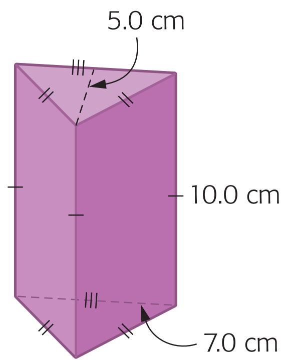
a)

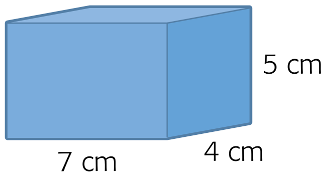


b)



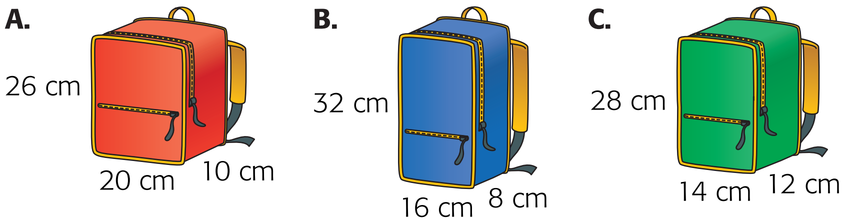
c)





5. a) Determine the volume of this prism.
   b) Triple the width, length, and height of the prism. What is the volume now?

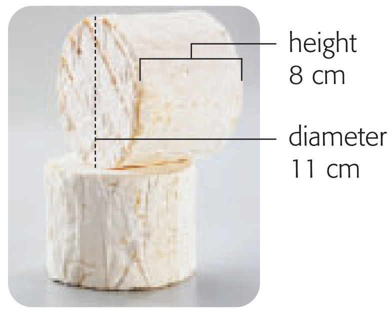
6. Which backpack holds the most?



7. Calculate the surface area and volume of each cylinder.

a)

b)



8. Icarus Airlines does not allow passengers to board an airplane with luggage that is more than 22 700 cm^3 in volume. Would a passenger be allowed to board an airplane with this suitcase? Explain.

9. A package of microwave popcorn is 8 cm wide, 10 cm long, and 1200 cm^3 in volume. The packages are shipped in boxes with a volume of 24 000 cm^3.
   a) How many packages of popcorn are in each box?
   b) Draw two different boxes that would hold the packages. Explain which box you would use.

What Do You Think Now?

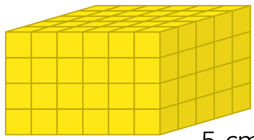
Revisit What Do You Think? on page 193. Have your answers and explanations changed?

Chapter Review

Frequently Asked Questions

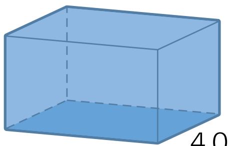
Q: How do you calculate the volume of a rectangular prism?

A1: You can model the prism using centimetre cubes.



This prism has 120 cubes, so its volume is 120 cm^3.

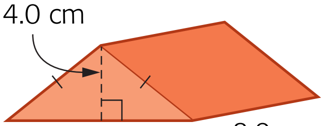
A2: You can multiply the area of the base by the height.



$$\begin{aligned} \text{Volume} &= \text{area of base} \times \text{height} \\ &= (10.0 \text{ cm} \times 4.0 \text{ cm}) \times 6.1 \text{ cm} \\ &= 244.0 \text{ cm}^3 \end{aligned}$$

Q: How do you calculate the volume of a triangular prism?

A1: You can divide the volume of a rectangular prism with the same width, length, and height by 2.



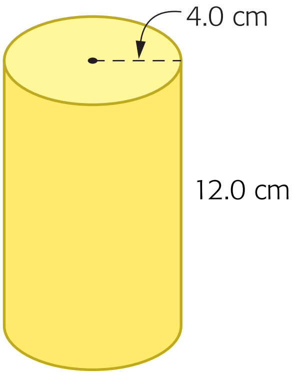
The volume of a rectangular prism 10.0 cm by 4.0 cm by 8.0 cm is 320.0 cm^3, so the volume of this triangular prism is 320.0 $cm^3 \div 2 =$ 160.0 cm^3.

A2: You can multiply the area of the base by the height.

$$\begin{aligned}\text{Volume} &= \text{area of base} \times \text{height}\\ &= [(10.0 \text{ cm} \times 4.0 \text{ cm}) \div 2] \times 8.0 \text{ cm}\\ &= 160.0 \text{ cm}^3\end{aligned}$$

Q: How do you calculate the volume of a cylinder?

A: You can multiply the area of the base by the height.



$$\begin{aligned}\text{Volume} &= \text{area of base} \times \text{height}\\ &= (\pi \times r \times r) \times h\\ &= \pi \times 4.0 \text{ cm} \times 4.0 \text{ cm} \times 12.0 \text{ cm}\\ &\doteq 603.2 \text{ cm}^3\end{aligned}$$

Practice

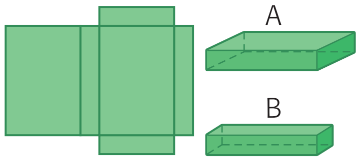
Lesson 5.2

1. Draw a net for each object.
   - **a)** a rectangular prism 8 cm by 5 cm by 3 cm
   - **b)** a cube with a side length of 6 cm
   - **c)** a prism 6 cm high with an isosceles triangular base 5 cm wide and 4 cm high
   - **d)** a cylinder 10 cm in diameter and 7 cm high

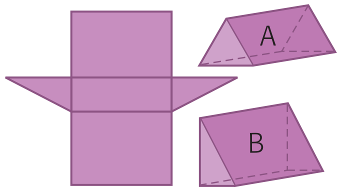
2. Explain how to determine the surface area of a rectangular box. Draw a net to support your explanation.

3. Match each net with its 3-D object. Explain your choice.

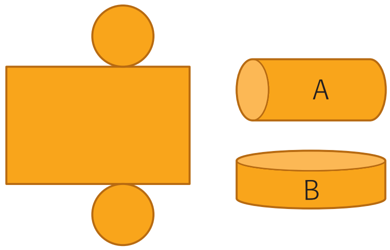
a)



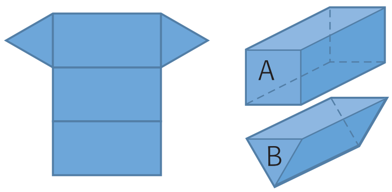
b)



c)



d)



Lesson 5.3

4. Sketch each object and label its dimensions.
Then calculate its surface area.

	Item	Length (cm)	Width of base (cm)	Height of base (cm)
a)	tissue box	22	7	10
b)	cereal box	16.3	5.0	27.5
c)	cheese in the shape of triangular prism	25.0	18.0	2.5

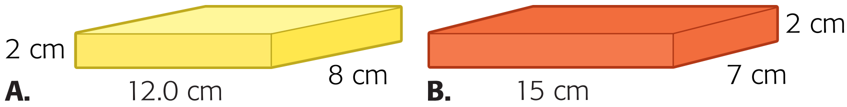
5. Ryan is making a cover for his hamster's cage. The cage is 80 cm long, 50 cm wide, and 40 cm high. How much material will he need, if he allows 5% more for the seams of the cover?

Lesson 5.4

6. How much waxed paper will Jake need to cover a cylindrical candle that is 6 cm in radius and 20 cm high?

Lesson 5.5

7. Jeanette is comparing two full boxes of the same kind of buttons at a store. Both boxes cost $2.99. Explain which box is the better buy.



8. An apartment building has a square entrance hall. There is a triangular planter in each corner of the hall. Each planter is 45 cm deep, and the two sides against the wall are each 90 cm long. What volume of soil is needed to fill all four of these planters?

Lesson 5.6

9. What might be the dimensions of a cylindrical container that contains 750 mL of yogurt?

10. Each week, the Fergusons put out one full round can of grass clippings for collection. The can is 50 cm in diameter and 65 cm high. What volume of grass do they put out each week?

Lesson 5.7

11. A company packages DVD collections in rectangular cases 20.0 cm high, 2 cm thick, and 600 cm^3 in volume. The cases are then packed into boxes for shipping.
    a) Draw and label two boxes of different dimensions that would hold 10 DVD collections that are packed tightly together.
    b) In which box would you ship the collections? Explain.

Chapter **Task**

Task | *Checklist*

- ✔ Did you explain each step of your calculations?
- ✔ Did you show all of your calculations?
- ✔ Did you explain your thinking?

Moving Day

You are moving and you want to pack all of your own special belongings. Some are very large, others are small.

? How much material and how much space would you need?

A. Select 10 items of different sizes and shapes to pack. Include items in the shape of rectangular prisms, triangular prisms, and cylinders.

B. Measure each item.

C. Write a description of each object and its dimensions.

D. Each box will hold one item. Draw a net for each box. Label the dimensions.

E. Calculate the amount of cardboard needed to make each box. Add 10% to allow for overlap.

F. Calculate the volume of each box.

G. Determine what percent of the moving truck your boxes will occupy.